Financial stability

Model risk in CLOs

Contact: damien.fennell@esma.europa.eu

Summary

The benefits of securitisation depend on its ability to effectively engineer and limit credit risk. This article explores the approaches to modelling CLO credit risk adopted by the three main CRAs. It discusses the differences and some limitations in approaches and how these might potentially affect credit ratings' accuracy. Finally, it sets the discussion in the context of some of the recent developments in the leveraged loan and CLO markets, including those stemming from COVID-19. Together, these make clear the importance of sensitivity analysis to identify model and credit rating limitations and how the transparency of these is key to informing investors' reliance on ratings.

Introduction

Structured finance promises benefits by creating lower risk securities from pools of higher risk collateral. Its rise in the 80's and 90's enabled borrowers to benefit from more plentiful and cheaper funding from investors who would not directly lend to them, but who were happy to invest in the structured lower-risk tranches.[73]

However, structured finance also presents risks. In the 2000's regulatory arbitrage, originate-to-distribute and an over-reliance on Credit Rating Agencies (CRAs) to assess credit risk resulted in the build-up of imbalances in the U.S. mortgage-backed Collateralised Debt Obligation (CDO) markets. The global financial crisis of 2007-2008 then made clear how a lack of due diligence by investors and conflict of interests among CRAs can result in dramatic effects on financial markets and the real economy (FCIC 2011).

In particular, the crisis showed how default correlation — a key input in securitisation — had been underestimated in credit rating models, leading to significant under-identification and under-pricing of risk. When house prices declined throughout the U.S., a large portion of mortgages that had been issued in different states and that had been pooled into mortgage-backed securities (MBSs) started to default at the same time. This resulted in waves of massive downgrades on AAA-rated CDOs and a rise in defaults for US CDOs.

Following the crisis, a European regulatory framework for CRAs was established[74] and a range of regulatory initiatives were taken to ensure that securitisation would better provide financing in the economy without jeopardising financial stability.[75]

The securitisation markets have changed since the crisis. Some of the worst performing products of the crisis, such as CDOs squared, have largely disappeared. However, in other areas structured finance markets have seen a resurgence. In particular, Collateralised Loan Obligation (CLOs) markets have raised concerns among policymakers.[76]

The recent surge in issuance of leveraged loans alongside a deterioration in underwriting standards and tight loan spreads (until the COVID-19 pandemic) has been fuelled in part by the rise of CLOs, which are estimated to account for about half of the leveraged loan market. Investors looking for yield have been attracted to

---

the relatively high returns and high credit ratings of most CLO tranches. Total leverage loans outstanding in Europe were about EUR 200bn in 3Q19, while CLOs outstanding stood at around EUR 120bn in September 2019.[77]

Given the recent growth in the CLO market and growing concerns, ESMA recently carried out a thematic review of the CRA methodologies for rating CLOs (ESMA 2020). The box below summarises its findings (RA.1).

RA.1
ESMA's thematic review of CRA CLO practices
Main findings of ESMA's CLO thematic review

The report's findings are as follows:

— *The internal organisation of CRAs* - the CLO rating process is segmented between a CLO analytical team and a corporate analytical team in all CRAs. A smooth and ongoing exchange of information between internal teams is key to ensuring a holistic assessment of CLO creditworthiness. CRAs should ensure the capacity for the timely identification of all inherent risks to CLOs;

— *The interactions with CLO issuers* - as CLO arrangers and managers can identify which CRA may assign the best ratings for each CLO tranche, it is key that CRAs ensure the independence of their rating process from any influence from their commercial teams and/or arrangers;

— *Model/third party dependencies leading to potential operational risks* - the dependency on rating models and data provided by third parties, and the high automation of processes, present operational risks that need to be monitored by CRAs to avoid potential errors in credit ratings;

— *Rating methodologies, modelling risks and commercial influence* - CLO methodologies are underpinned by assumptions and modelling approaches that can have an impact on credit ratings. ESMA highlights the importance of providing transparency to market participants on the limitations of methodological approaches. In addition, CRAs should ensure that evolutions in CLO methodologies are not influenced by commercial interests;

— *The thorough analysis of CLOs* - it is key that CRAs continue to monitor market trends and perform a thorough analysis of all relevant developments in CLO contractual arrangements.

As the report is based on information collected up until March 2020, it does not assess the consequences of the COVID-19 outbreak. In light of this, ESMA expects CRAs to continue to perform regular stress-testing simulations and provide market participants with granular information on the sensitivity of CLO credit ratings to key economic variables.

This paper was informed by and complements the thematic review, by focusing on the type of models used by CRAs to assign CLO ratings. In addition, while informed by the thematic review, the paper is based on publicly available information. It does not disclose or rely on information shared with ESMA by the CRAs as part of the thematic review or other supervisory activities.

The paper aims to understand if the lessons of the financial crisis have been sufficiently learnt – given the past experience of CDOs and the recent growth of CLOs – or whether the benign macroeconomic backdrop (until the COVID-19 pandemic) has increased risks for the next crisis.

To answer this question, we take a closer look at the type of models used by CRAs to assess default risk among CLO tranches and assign credit ratings. We do not investigate, assess or compare specific CRAs' CLO rating models or processes. Instead, the aim is to identify the kinds of model risk that can arise for the type of models used by CRAs.

We show that the modelling and calibration of default correlation within the CLO portfolio is key in determining credit ratings. However, owing to a lack of data the estimation of correlation is very difficult. Nonetheless, moderate changes in default correlation can have a sizeable impact on default probability (and on credit ratings' accuracy). This in turn underlines the importance of model sensitivity analysis and stress testing. Yet, as found in the thematic review, reverse stress tests and sensitivity analysis of default correlation among the three largest CRAs remains limited.[78]

Moreover, recent developments in the leveraged loan market point to a deterioration in loan documentation, a widespread decline of financial covenants and increased use of accounting techniques by borrowers to reduce their apparent financial leverage (i.e. 'add-backs' influencing EBITDA levels).[79] These trends magnify the risk that when defaults occur, they are more likely to occur together – clustered across firms and sectors (since it takes a higher shock to make firms default).

The remainder of the paper is structured as follows: the next section provides an overview of

---

77 Leverage loan market size from AFME (2019), CLO market size from TwentyFourAM (2019).

78 ESMA (2020).

79 See, for example, p.7-10, FSB (2019).

the main model types used by CRAs. We then compare these before exploring some limitations of the models using sensitivity analyses, in particular looking at the modelling of tail-dependence in defaults. It then looks at the relevance of this model risk in the context of the ongoing COVID-19 crisis. The paper concludes with a call for more transparency on CRA models and their limitations, including for more information on the modelling of tail-dependence of defaults, which tend to cluster during recessions.

What are CLOs?

A CLO is a securitised product backed by a pool of leveraged loans on the asset side, which are funded by the issuance of debt and equity CLO tranches with a different degree of seniority on the liability side. The interest and principal payments on the loans are repaid to the tranche investors according to a waterfall approach. The most senior AAA-tranches are paid first, then mezzanine tranches and so on. Once the debt tranches have been paid, equity tranches are paid the remaining revenue. When losses occur, equity tranches are the first to absorb losses, then the most junior debt tranches and so on. Senior tranches are protected unless the losses are too large to be absorbed by the equity and mezzanine tranches (RA.2).

Total portfolio defaults and losses depend on how likely defaults are to co-occur among the loans in the pool. When loans tend to default together (i.e. when default correlation is higher) then the total loss in the portfolio will tend to be higher and more senior tranches will be more likely to default. In addition, the more correlated defaults are, the better this is for the equity and junior tranches and the worse it is for the more senior tranches. This is because when defaults co-occur then the total loss (when it occurs) will be higher, and the losses are more likely to be large enough to exceed the junior tranches and any overcollateralisation, and thus hit the more senior tranches.[80]

RA.2
The structure of a CLO
Tranches, risk, returns and payment waterfall
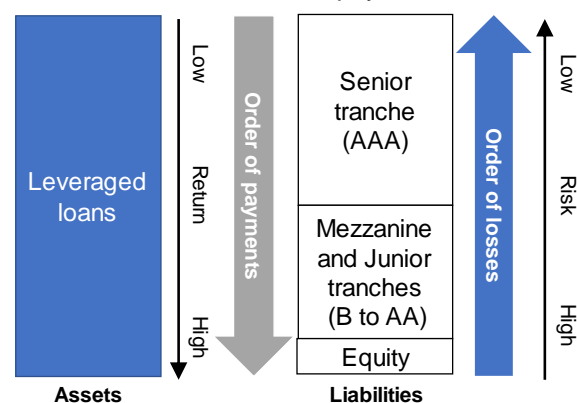
Source: Natixis Asset Management (2017) with ESMA amendments

Unlike other securitised products, in CLOs the pool of loans is usually dynamic and actively managed.[81] CLOs also have covenants that stipulate how the CLO should be managed. The CLO manager runs the CLO and is usually able, within constraints set by the covenant, to buy or sell loans in the underlying pool. This flexibility typically lasts until principal repayments begin to be made to the debt tranches, as the underlying loans themselves are repaid. This ability to manage the underlying pool of loans makes the CLO manager important to the performance of the CLO.

Included in the CLO covenants are a range of tests that act as regular checks to ensure a CLO works as designed. These include the overcollateralisation tests (OC) for the debt tranches. For a given debt tranche, the OC test checks if the value of the pool of loans, less the face value of more junior tranches, remains higher than the face value of the tranche. If an OC test fails, all excess cash flows are diverted to the senior tranches. Another test, the interest coverage test, checks if the total interest due on the leveraged loans is sufficient to cover the interest owing to CLO investors. Failure in any of the test triggers some restrictions on the CLO manager in order to protect senior noteholders.

When a CLO is created, an arranger – typically an investment bank – advises the issuer on the structure of the CLO and a credit rating agency rates the tranches. In the EEA issuers are

---

[80] In other words, with higher correlation expected losses tend to be shared more among the different tranches. As a result, more senior tranches will tend to fare worse on average and more junior tranches will tend to fare better when default correlation is higher.

[81] CLOs with actively managed portfolios do not to qualify for STS status under the Securitisation Regulation. See Article 20(7) of the Securities Regulation.

required to obtain two credit ratings for each structured finance debt instrument issued, which needs to be from different and independent CRAs.[82] In addition to issuing an initial rating for a debt tranche, the CRA monitors the performance of the CLO debt tranche through the life of the CLO and may take a rating action (issue a rating outlook, watch, downgrade or upgrade) as circumstances change, for example, if the credit quality of the underlying loans changes.

CRA models for rating CLOs

CRAs models for rating CLO debt tranches can be typically split into two parts:

— *A portfolio model* which assesses the overall credit risk of the pool of leveraged loans, using inputs on the individual loans and the portfolio.
— *A cash-flow model* which assesses tranche payments, defaults and losses that would arise for different performance outcomes for the pool of underlying loans.

This can be used to evaluate default probabilities and expected losses for CLO tranches. These two models together enable CRAs to assess the likelihood of defaults and losses to the different debt tranches and to inform their assignment of credit ratings.

The Gaussian copula approach

This section describes in a simplified way the main modelling approach used by CRAs. In Gaussian copula portfolio credit models, loan defaults are assumed to follow a Gaussian copula, the copula associated with multi-variate normal distributions.[83]

The models that use a Gaussian copula approach estimate the distributions of defaults and losses for the pool of leverage loans using the Monte-Carlo simulation technique.[84] In simplified terms, these models simulate the loan performances in the portfolio using information

about the composition of the portfolio and inputs on the individual loans (default rates, expected recoveries) and correlations among loan defaults. The individual loans' performances form a scenario that demonstrates how the whole portfolio might perform. By repeating this simulation a large number of times, one can create the distribution of possible portfolio outcomes. From this the likelihood of different defaults and losses occurring in the portfolio can be estimated.

The cash-flow models are then used to estimate the distribution of payments, defaults and losses for the different CLO tranches. A credit rating for each tranche is then assigned based on where that tranche's default probability or expected losses sit in tables mapping these to rating categories.[85]

While the description above captures the core of how some CRAs assign CLO tranche ratings, in practice the rating process is significantly more complex.[86] The assignment of credit ratings is not determined simply by the application of a model. There are more steps, including feeding in qualitative information that informs the rating. Moreover, the CRAs typically describe their ratings as opinions on the relative rankings of credit worthiness rather than strict measures of default probability or expected loss.

CRAs also usually introduce additional stresses when constructing ratings using a Gaussian copula approach. To mention just a couple, S&P, for example, requires that a rating is stable under the default of the largest obligor, and for AAA and AA-rated tranches it requires that it is stable under the default of the largest industry in the pool.[87] However, Fitch adds the 'obligor concentration uplift', to add conservatism to the rating when a portfolio has loans concentrated in a small number of obligors.[88]

As mentioned earlier, total portfolio defaults and losses depend on how likely defaults are to co-occur among the loan pool. When loans tend to default together (i.e. if default correlation is

---

[82]   See Article 8c(1) of the CRA Regulation.

[83]   Mathematically, a copula is a function that maps the univariate distribution functions of a set of jointly distributed random variables to their multivariate distribution function. It enables the modelling of the joint dependence of those variables separately from their individual behaviours. The Gaussian copula is the copula of multivariate normal distributions.

[84]   See, for example, S&P's 'CDO Evaluator' (p.3, S&P, 2019) and Fitch's 'Portfolio Credit Model' (p.2, Fitch, 2019).

[85]   See for example Fitch (2019) p.31 and p.36 and Moody's (2018).

[86]   See ESMA (2020) for a more detail on how CRAs rate CLOs.

[87]   See p.11-13, S&P (2019).

[88]   See p.13-14, Fitch (2019).

higher) then the total loss in the CLO portfolio will tend to be higher and more senior tranches will be more likely to default. Thus, how well a portfolio credit model captures the co-occurrence of loan defaults is central to its ability to model the credit risk of the whole loan portfolio and of the CLO debt tranches in turn.

The Gaussian copula has mathematical advantages but captures loan default co-occurrence in a limited way. It assumes that loan defaults do not exhibit any 'tail-dependence', that is, how loan defaults happen together does not vary with the extremity of the situation. Mathematically, it assumes a constant default correlation between loans. This means that it treats the likelihood of an occurrence of simultaneous default during a recession the same as it would in normal times. CRA models often also introduce further simplifications, for example, assuming that this default correlation between any two loans is constant over time and is determined by a few features, such as whether or not the loans are from the same industry and/or geographical region.[89]

The Binomial expansion technique

Another approach to modelling portfolio credit risk uses the binomial expansion technique (BET). This approach approximates the behaviour of the actual loan portfolio, if loan defaults are usually correlated, with a hypothetical portfolio of loans in which loan defaults are not correlated and whose defaults follow a binomial distribution.[90]

Using the properties of the binomial distribution, the default and loss probabilities of the loan portfolio can be straightforwardly calculated. The cash-flow model is then used to estimate the default probabilities and expected losses for the CLO tranches. CLO ratings can then be assigned by mapping these losses to ratings.

Similar to Gaussian copula models, CRAs that use the BET model also incorporate refinements to make their ratings more robust. Moody's, for

example, introduces a default probability stress factor that effectively raises the rating cut-offs for these more senior ratings by applying a stress to the underlying loan default probabilities when calculating ratings. The default probability stress factor applies if the target rating of a tranche is associated with a low default probability. In these cases, a stress to increase the default probability of the underlying loans is applied, and the tranche ratings are calculated under this stress.[91]

When it was developed, the BET had a major advantage because default probabilities and expected losses of the loan portfolio could be calculated from it analytically, without the numerous calculations of the Monte-Carlo approach which were then slow to compute. With much more powerful computing now widely available, this advantage has been reduced.[92] We nonetheless discuss the BET model here because it is used as part of the assignment of some CLO ratings.[93]

The core of the BET is the modelling of a correlated loan pool using a hypothetical pool of uncorrelated loans. To make it representative of the actual portfolio, the defaults of the hypothetical portfolio are assumed to have the same mean and variance as the actual portfolio (1st and 2nd moment matching). For simplicity, loans in the hypothetical portfolio are assumed to have equal weight. On that basis, one can then derive a relationship between the number of loans in the hypothetical portfolio, which is called the 'diversity score', and the default correlation between loans in the actual portfolio.[94] This shows that the diversity score implicitly captures the default correlation of the actual loan portfolio. The relationship is an inverse one, a portfolio with a higher (lower) diversity score has lower (higher) default correlations. In addition, as the diversity score is rounded down to a whole number when calculated, it implicitly assumes a higher correlation than the correlation from which it is calculated. (RA.7)

The diversity score depends on the number of loans in the portfolio and how many of these are

---

[89] See, for example, p.8-10 Fitch (2019) and

[90] The BET approach was developed by Moody's. See Cifuentes et al. (1996) for the original paper setting out the BET approach.

[91] The most severe default probability stress is applied where tranche ratings are expected to have very low default probabilities (that is a target rating of Aaa) In this case the underlying loan default probabilities are almost doubled (scaled up by 1.95). As the stress applies to tranches that have a low default probability, it affects

higher-rated tranches. Lower rated tranches (B1 or lower) do not face a stress. See p.35, Moody's (2019b).

[92] Moody's now complement their BET model with a Monte-Carlo simulation model that uses a Gaussian copula (CDOROM). See p.59, Moody's (2019a).

[93] See p.9, Moody's (2019b).

[94] For further discussion of the diversity score and how it is derived, see p.271-5 in Bluhm et al. (2002), p.3-5 in Fender et al. (2004), and p.471 in Nickerson et al. (2017).

in the same industries.[95] Diversity scores contributions can vary also depending on geographical regions.[96] Different ways to calculate the diversity score implicitly make different assumptions about loan correlations in the actual portfolio. Similar to CRAs using the Gaussian copula in modelling correlations, how the diversity score is calculated can incorporate important assumptions, for example, the assumption that the defaults of loans in different industries are uncorrelated.

Model calibration

This section briefly outlines how the CRAs set certain key inputs to the model. Model calibration is discussed in more depth in the recent thematic review of CRA practices carried out by ESMA.

Correlation (and diversity score) inputs are particularly important given their importance in assessing portfolio credit risk. Default correlation, however, is also inherently difficult to measure given that it tends to shift in crisis periods, crisis periods tend to be distinctive from each other, and data on crisis periods are relatively limited compared with data from other periods.[97]

CRAs use credit ratings for the underlying leveraged loans (and sometimes credit opinions if ratings are unavailable) as the key inputs for modelling the individual loans in their portfolio models. These are based on historical data linking credit ratings to observed default probabilities for the loans in the portfolio based on their characteristics.[98] Recovery rates are also input based on tables that have been calibrated to historical recovery data.[99]

In setting levels for tranche ratings in terms of default probabilities or expected losses, some CRAs require their highest-rated tranches to be such that they can withstand historically high levels of defaults.[100] Some CRAs use the mappings used for non-structured finance assets to assign ratings in terms of default probabilities or expected losses.[101]

To calibrate correlations, CRAs tend to use a combination of simplifying assumptions (i.e. the correlations take particular values depending on the industries and regions of the loans) with indirect calibration to historical data.[102]

Model sensitivities

This section analyses how the choice of model when rating CLOs can affect ratings. We construct simple copula and BET models and simulate how their outputs vary under different conditions. The aim is to identify similarities and differences among the model types, to identify sensitivities that these kinds of models have, and to understand what insights these may yield for CLO ratings, particularly in stress situations.

It is important to note that we are not comparing specific CRA models here. The aim is to instead understand how models of the type used by CRAs can give different results as inputs change. No particular model discussed here should be associated with a particular CRA, as none of the models here capture the calibration or the more detailed steps and processes used by CRAs to assign ratings.

The work extends on the analysis in TRV 2-2019, which analysed how CLO ratings could vary with correlations and the choice of copula (ESMA, 2019).[103] As was done there, in our modelling we assume a CLO composed of characteristics in the table below (RA.3). Unless otherwise stated, or being varied in the simulation, this presents the CLO modelled in the simulations. The idealised CLO has 100 leveraged loans, each with the probability of default of 20%, corresponding to the five-year default probability of B-rated loans.

The CLO structure is divided into four tranches (equity, junior, mezzanine, senior). The equity tranche absorbs up to the first 8% of losses, followed by the junior tranche (up to 20%), the mezzanine tranche (up to 40%) and finally the senior tranche. So, if losses reached 10%, the equity tranche would be wiped-out, and the junior tranche would absorb the remaining 2%. For simplicity, we do not consider prepayment or interest rate risk in the model in order to focus mainly on correlation. For simplicity, there is also

---

[95] See, for example, p.39-40, Moody's (2019b).

[96] See, for example, p.43-6, Moody's (2019b).

[97] Difficulties in and different approaches to measuring correlation as discussed in Nickerson et al. (2017).

[98] See, for example, p.17-26 S&P (2019) and p.35-7, Fitch (2019).

[99] See p.9, S&P (2019) for example.

[100] See S&P (2019) for example.

[101] See both Moody's (2018) and Fitch (2019) for example.

[102] Fitch, for example, calibrates correlation by matching the default rates of modelled portfolios to the historically observed rates. See Fitch (2019).

[103] ESMA (2020).

no overcollateralisation assumed for the CLO (if the face value of the leveraged loans is higher than CLO liabilities).

RA.3
Loan and CLO assumptions in simulation analysis

Portfolio characteristics

Loans	100 loans (equal par with zero coupon)
	(10 loans each in 10 different sectors)
Default probability	20% over five years
Default correlation	0.2 - loans in same sector
	0 - loans in different sectors
Recovery rate on default	50%
Maturity	5 years

CLO structure

Equity	8% of portfolio
Junior	12% of portfolio
Mezzanine	20% of portfolio
Senior	60% of portfolio

Note: Assumes fixed recovery rate in case of default, no ramp-up or wind-down period, no changes in loan portfolio over life of CLO.

Copulas were modelled using Monte-Carlo simulation with at least 100,000 runs. To run the BET simulations, the diversity score of the portfolio was first calculated from the correlations and the number of loans in the portfolio,[104] the binomial distribution was then used to analytically model outcomes. Finally, for 'the stressed BET', we applied a stress factor[105] to scale up the underlying loan probabilities when it was required for a tranche. As the stress depends on the rating targeted by a tranche, the target rating was taken to be the highest rating for the tranche that did not fall under the stress (to capture the best rating that was robust under the stress).[106]
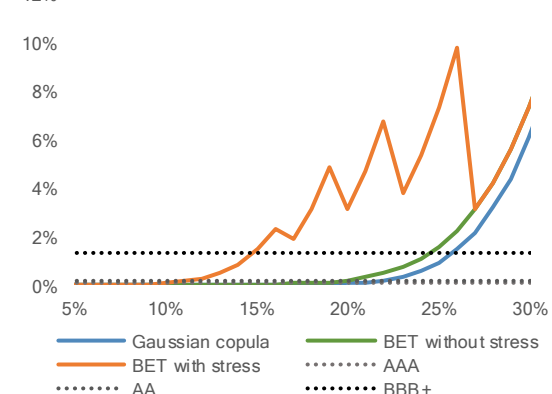
Default probabilities and recoveries

First, we consider how ratings may change as the underlying loan default probability or recovery rates change. The results show, as one would expect, that credit ratings generally fall as the credit quality of the underlying loans falls (RA.4) or as recoveries fall (RA.5). It is noteworthy in these charts (and others that follow) that the default probabilities show significant oscillations under the stressed BET, this is an artefact of the stepped nature of the stress factors, which in some cases as the unstressed default probability of the tranche increases, is subject to a weaker stress which means that the probability of defaults for the tranche under the stress actually falls. Given this, when interpreting results of the stressed BET in this paper, we disregard these oscillations and focus instead on the trend.

RA.4
Default tranche probability and loan defaults
Deteriorating loan quality can drive sharp rises

Note: Simulated mezzanine tranche default probability (y-axis) under different assumed default probabilities of the underlying loans (x-axis). Assumes 50% recovery rate and uses Fitch tables for ratings levels.
Sources: ESMA calculations.

All three models show a rapid increase in tranche default probability (here the mezzanine tranche) beyond a certain point. The stressed BET is by far the more conservative. In our simulated CLO, it would rate the mezzanine tranche BBB+ when the underlying loans had 15% default probability whereas both the (unstressed) BET and Gaussian models would only rate the tranche this

---

[104] Expressions for diversity score were derived using 1st and 2nd moment-matching both in the case of a portfolio with a flat pairwise default correlation across all loans, and in the case where there are different correlations for loans in the same and in different sectors. This generalised the derivation on p.471 in Nickerson et al. (2017), assuming different pairwise default correlations for loans in the same and different sectors.

[105] Here we use Moody's table of stresses for simplicity. See p.35, Moody's (2019).

[106] As stress factors above 1 only apply when tranche default probabilities are low, senior and mezzanine tranches are the only ones affected by the stresses in the results below, for equity and junior tranches, stress BET results were the same as the unstressed BET.

level if the underlying loans had a default probability of at least 25%.

For variation in recovery rates the picture is similar – default probabilities rise rapidly under all models once recoveries fall low enough. Stressed BET is again significantly more conservative than the other two models (RA.5).

These two charts show just how much the stress in the BET can affect the tranche rating. This effect is owing to the stress factors being high when the unstressed tranche default probabilities are low. At their highest the stress factors almost double the underlying loan default probability, which has a large impact on the expected loss and default probability of the CLO tranche.

RA.5
Default tranche probability and recovery rates
Lower recoveries can sharply increase risk
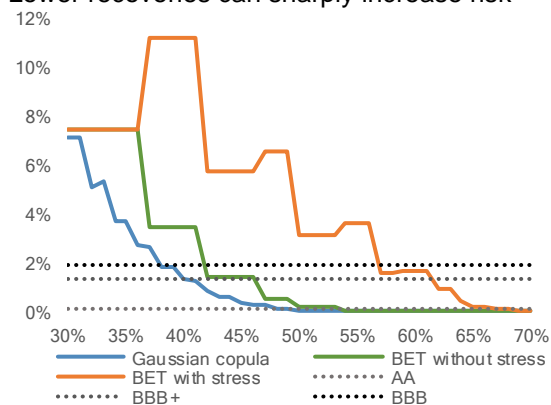
Note: Simulated senior tranche default probability (y-axis) under different assumed default probabilities of the underlying loans (x-axis). Assumes 20% probability of default for underlying loans, rating levels calibrated from Fitch tables.
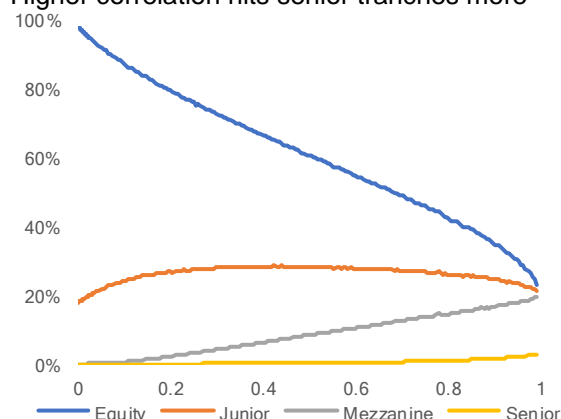Sources: ESMA calculations.

Correlation

As mentioned above, correlation is a key input to CLO rating portfolio models. Here we look at how changing correlation can affect CLO ratings. The analysis extends that which is presented in TRV 2-2019, because it allows default correlations to differ depending on whether loans are in the same sector or in different sectors. Distinguishing inter-sector and intra-sector correlation in this way is also closer to how the CRAs model correlation for rating CLOs, as discussed earlier.

First, we explore the simpler case in which the default correlation is assumed to be the same for all loans, first for the Gaussian copula (RA.6) and then for the stressed BET (RA.7). Varying loan correlation from totally uncorrelated to fully correlated, the charts show how expected losses for the tranches change as correlation increases. For the equity tranche, expected loss falls as

correlation rises, reflecting that fact that when loan defaults co-occur, losses are more likely to exceed the equity tranche and also be borne by the other tranches. More senior tranches thus fare worse as correlation rises, while the impacts on the junior tranche fall in between those of the equity tranche and the more senior tranches.
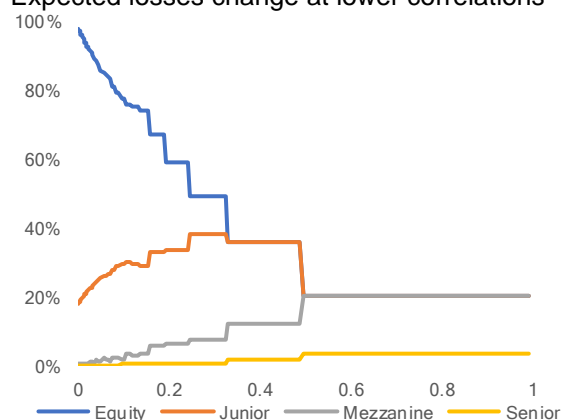
RA.6
Expected loss and default correlation - Gaussian
Higher correlation hits senior tranches more

Note: Gaussian copula tranche expected losses as proportion of tranche value in % (y-axis) for different default correlation values (x-axis) - assuming same default correlation among all loans.
Sources: ESMA calculations

The stressed BET model exhibits the same broad pattern of the Gaussian copula, but with a clear difference. In the BET the expected losses of the tranches change in steps, and these changes occur at a lower correlation than in the Gaussian model, which shows it is more conservative in assigning default probability to the more senior tranches (RA.7)

RA.7
Expected loss and default correlation – stressed BET
Expected losses change at lower correlations

Note: Stressed BET tranche expected losses as proportion of tranche value in % (y-axis) for different default correlation values (x-axis) - assuming same default correlation among all loans.
Sources: ESMA calculations

This difference is due to the way in which how the BET models the distribution using a hypothetical

portfolio, whose number of assets equals the diversity score. The diversity score is always a whole number and falls in unit steps with increasing correlation. In addition, as it is rounded down it implicitly and conservatively assumes a higher correlation for the loan pool. These also create step effects, as explained in the box below (RA.8).


RA.8
Diversity score in the BET model
Step effects from diversity score changes

In the BET model, the diversity score is the number of assets in the hypothetical portfolio used to model outcomes for the actual portfolio. As such, it is always a whole number. So, while the diversity score falls with increasing correlation, it can only fall in unit steps, as the chart RA.9 illustrates for our idealised CLO.

RA.9
Diversity score and default correlation
Diversity score falls in steps as correlation rises
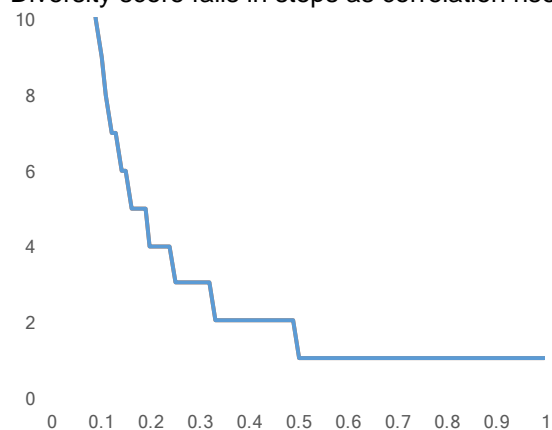
Note: Diversity score (y-axis) vs correlation (x-axis) for the idealised CLO, assuming the same default correlation between all loans.
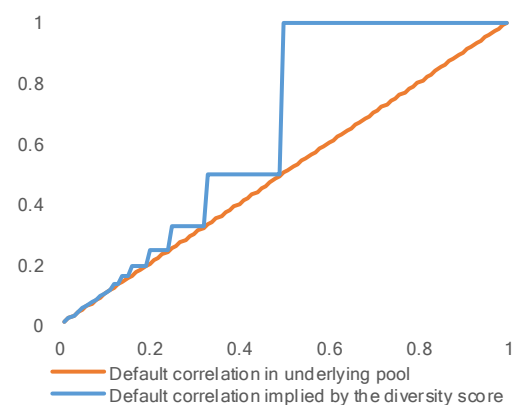Sources: ESMA calculations

The step changes in the diversity score mean that when it is low, a diversity score change can significantly change the BET's binomial distribution, which can lead to large step changes in modelled outcomes (see, for example, the step changes in expected loss in RA.7 for correlations above 0.2).

In addition, when calculating the diversity score from the correlation it is rounded down to nearest whole number. This increases the correlation assumed for the loan pool. Chart RA.10 shows how the correlation implicitly assumed by a diversity score varies with the correlation of the loan pool used to calculate the diversity score, for the idealised CLO.

RA.10
Diversity score and default correlation
Implicitly assumes steps in loan correlations

Default correlation in underlying pool
Default correlation implied by the diversity score

Note: Correlation implied by the diversity score (y-axis) vs correlation of the underlying pool used for calculating the diversity score (x-axis) for the idealised CLO used in our modelling.
Sources: ESMA calculations

The extent of rounding up of correlation increases as the underlying loan correlation increases. For low correlations impacts are minimal, but for high correlations it becomes significant. In our model CLO, for example, once the default loan correlation reaches 0.5 or higher, it is treated as one in the BET model.


Because of these features of the BET model, the expected loss changes in the BET model are fully realised once the default correlation reaches 0.5 because at this point the diversity score is one, which is equivalent to assuming a default correlation of one. For correlations above 0.5, the BET models no longer model any differences. While this shows that the model is conservative in its treatment of correlation when correlation is high, it also shows a limitation of the BET model, namely, that it becomes less discriminating between different outcomes as correlation rises. Overall, the BET model works best when correlation is low. When correlation is high it becomes much more conservative.

We now explore the more complex situation, in which intra-sector and inter-sector loan correlation can take different values. This is more realistic and closer to CRA models. The charts below show how increasing inter-sector correlation affects the senior and mezzanine tranches respectively, for three values of intra-sector correlation – 0.15, 0.2 and 0.25. (RA.11, RA.12).

RA.11
Senior tranche defaults and sector correlations
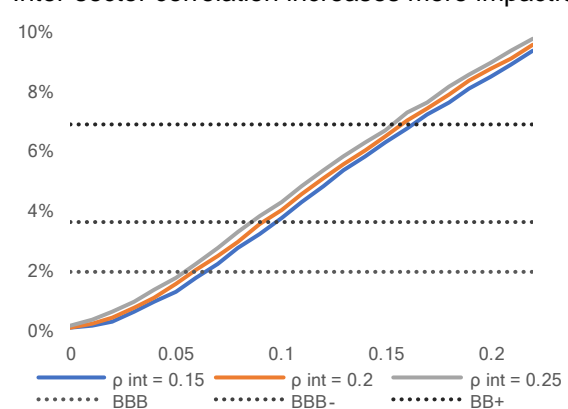## Sector correlations increases can affect ratings
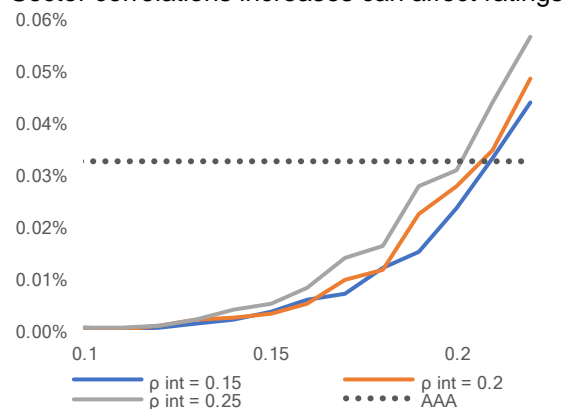
Default senior tranche probabilities (y-axis) vs inter-sector correlations (x-axis) for different values of intra-sector correlation (0.15, 0.2 and 0.25). Rating levels from Fitch tables.
Sources: ESMA calculations

RA.12
Mezzanine tranche defaults and sector correlations
Inter-sector correlation increases more impactful

Note: Default mezzanine tranche probabilities (y-axis) vs inter-sector correlations (x-axis) for different values of intra-sector correlation (0.15, 0.2 and 0.25). Rating levels from Fitch tables.
Sources: ESMA calculations

For the senior tranche, an increase in intra-sector correlation increases the default probability a little, but much less than does an increase in the inter-sector correlation. For example, an increase in inter-sector correlation from 0.15 to 0.2 triples the default probability, while the same increase for intra-sector correlation at most increases the default probability by less than half. A similar pattern is visible for the mezzanine tranche – here a 0.05 increase in intra-sector correlation increases default probability by less than 0.5%, while the same increase in inter-sector correlation increases the default probability by over 2%.

The mathematical reason for this is that in our underlying portfolio there are many more loans in different sectors than there are in common sectors. Increasing correlation among loans in different sectors therefore has a much larger impact on portfolio risk and default tranche probabilities. In practice, this shows that there is a model vulnerability (also shared by the BET models) to increases in correlation, and not only large increases in correlation (visible in all of the charts here) but also small increases in default correlation for a large number of loans.

Tail dependence

To conclude this section, we perform a similar analysis to that carried out in TRV 2-2019 looking at how the modelling of the tail of the portfolio risk distribution is important to ratings. In this case we also incorporate the BET models to see how they compare.

The chart below compares how the default probability of the mezzanine tranche, as modelled under different copulas (Gaussian, T, Clayton) and the two BET models, varies with loan default correlation.[107] To simplify the analysis, we assume that all of the loans share the same default correlation. (RA.13)
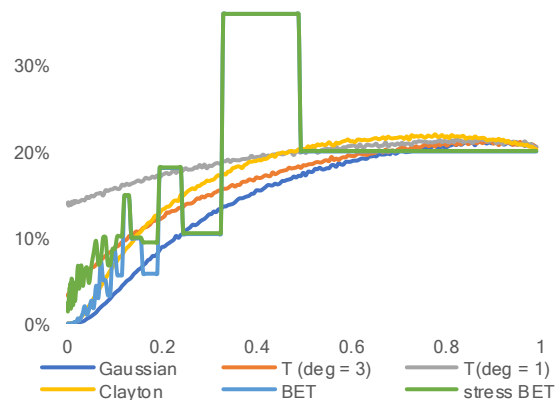
---

[107] As the Clayton copula is not parametrised by correlation, a Kendall tau is calibrated from the correlation value using a standard formula.

RA.13
Different copulas and BET
Stressing BET fattens the tail for low correlations

Note:Mezzanine tranche default probabilities (y-axis) vs. correlation (x-axis), assuming constant correlation across all loans.
Sources: ESMA calculations

Default probability increases with increasing correlation under all models. In line with TRV 2-2019, there is also a big difference in the modelled default probability across the copulas. As the tranche default probability increases with increasing correlation, the Clayton copula becomes the most conservative of the copula models in its assessment of default probability. This reflects its ability to model increased tail dependence for the right-hand tail of the default distribution (i.e. more stressed scenarios increasing default correlation, increasing the risk of multiple defaults in the pool). The T distributions also fatten tails (and are more conservative across correlation values), while the Gaussian copula is the most 'optimistic' giving the lowest default probability of the copulas at all correlation values.

The BET models are generally more conservative than the Gaussian. We can also see how the addition of the stress to the BET fattens the tail when correlations are low. For low correlations it gives much higher probabilities of default than the BET. Again, we see the limits of the BET at high correlation values, with increasingly large jumps as correlation rises, before modelled probabilities become constant once correlation exceeds 0.5.

Relevance

Possible implications for CLO ratings

The COVID-19 situation has had wide-ranging impacts on the real economy and financial markets, leading to a general deterioration in economic outlook. It has acted as a major shock

to credit risk which has led to widespread downgrades in corporate debt, including in leveraged loans, which have started to be downgraded and put on negative watch or outlook. (RA.14).

RA.14
Leveraged loan downgrades
EU downgrades accumulate in early 2020
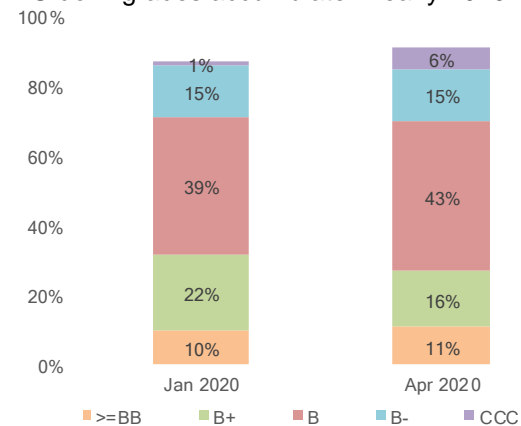
Note: Does not add to 100% due to median calculation and unreported ratings
Sources: Reproduced from Barclays Research (2020), which used data from Kanerai, Intex and Barclays Research.

Deteriorating leveraged loan performance and credit quality directly links with analyses that we have carried out. In particular, our analysis is directly relevant to questions that interest investors and regulators are asking at this time, such as, to what extent leveraged loan rating downgrades (which correlate with loan default probabilities) will lead to downgrades in CLO tranches, and which tranches will be affected and by how much.

There are several links between the analyses above and the COVID-19 context. First, increasing the risk of leveraged loan downgrades is indicative of increasing underlying loan default probability. Chart RA.4 is thus relevant here. It shows that all of the models would increase tranche default risk as the underlying loan credit quality falls. The stressed BET would be more conservative from the outset and it would downgrade tranches sooner than the BET or the Gaussian copula. RA.15 presents this explicitly in terms of ratings. It shows how each model would re-rate the idealised CLO as loan default probability increases.

RA.15
CLO rating sensitivity to loan credit quality deterioration
Mezzanine tranche sensitivity to loan downgrades

Loan default probability (rating)	20% (B+)	22% (B)	24% (B)	26% (B)	28% (B)	30% (B)
Gaussian copula	AAA	AA	A+	BBB+	BBB	BBB−
BET	AA	A+	A−	BBB	BBB−	BB+
Stressed BET	BBB	BBB−	BBB−	BB+[108]	BBB−	BB+

Note: Calculated for the idealised CLO, with loan recovery rate of 50%, intra-sector default correlation of 0.2 and zero inter-sector correlation. Ratings assigned using Fitch rating tables.

This type of analysis provides an answer – in the case of our idealised CLO under the simple models used – to the question of how much might a CLO (here the mezzanine) tranche be affected as the average credit quality of loans in the portfolio deteriorates. Interestingly, the sensitivity of tranche default probability appears relatively high under all three models. It takes only an increase in default loan probability from 20% to 22% to lead to a tranche downgrade.

The table below presents a similar analysis, but for recovery rates (RA.16). It shows that lower recovery rates on underlying loans can also lower CLO ratings, with small falls in expected recovery rates when loans default (e.g. 2% falls) leading to downgrades across the three models.

RA.16
CLO rating sensitivity to lower recovery rate
Mezzanine tranche sensitivity to lower recoveries

Loan recovery rate	50%	48%	46%	44%	42%	40%
Gaussian copula	AAA	AA+	AA−	A	A	BBB+
BET	AA	A+	BBB+	BBB+	BBB+	BBB
Stressed BET	BBB	BBB−	BBB−	BBB−	BBB−	BB

Note: Calculated for the idealised CLO, with underlying loan default probability of 20%, intra-sector default correlation of 0.2 and zero inter-sector correlation. Ratings assigned using Fitch rating tables.

This is particularly relevant in the current context given concerns about 'cov-lite' leverage loans. While the weaker covenants of cov-lite loans may delay the triggering of a loan default, it may worsen recoveries when the default actually occurs (as the default may occur at a point when the loan has deteriorated further). If cov-lite loans' recovery rates are lower than anticipated, then CLO tranche ratings may be affected.

Next, we look at how increases in inter-sector correlation could affect CLO ratings in our idealised model (RA.17). Here we see, in line with RA.12, that very small increases in inter-sector correlation can also lead to rating downgrades for the mezzanine tranche. This has particular relevance in the context of COVID-19 given that we have seen severe economic impacts across sectors that would not normally be expected to be so correlated. Given this, there may well be increases in default correlations for some loans in different sectors which could contribute to CLO tranche downgrades.

RA.17
CLO rating sensitivity to increased inter-sector correlation
Mezzanine tranche sensitivity to correlation

Inter-sector correlation	0	0.01	0.02	0.03	0.04	0.05
Gaussian copula	AAA	AA	AA−	A	A−	BBB+
BET	AA	A−	BBB+	BBB+	BBB+	BBB
Stressed BET	BBB	BBB−	BBB−	BBB−	BB+	BBB−

Note: Calculated for the idealised CLO, with underlying loan default probability of 20%, recovery rate of 50% and intra-sector default correlation of 0.2. Ratings assigned using Fitch rating tables.

Another issue, not explicitly captured above, is how CLO ratings might be affected by multiple parameter shifts. In the COVID-19 crisis, we could see a combination of deteriorating leveraged loan credit quality, lower-than-expected recovery rates, and increases in default correlations. Occurring together, these would have a cumulative impact which could not only lead to more significant and numerous downgrades of CLO tranches, particularly for the junior and mezzanine tranches, but also affect some senior tranches.

In addition, the different models (Gaussian, BET and stressed BET) often rate the same CLO tranche differently. This highlights the risk of the model chosen not capturing the evolving credit dynamics of the COVID-19 crisis well. For

---

[108] Note that the counterintuitive rating upgrade under the stressed BET model when the loan default probability increases from 26% and 28% is driven by the stress factor falling as the unstressed BET rating falls, which lowers the underlying probability of the loans used to calculate the rating under the stressed BET model. A similar increase is seen in the correlation sensitivity table when the inter-sector correlation increases from 0.04 to 0.05.

example, the stressed BET by adding stresses to the senior and mezzanine tranche might better capture the 'fattening of tails' that can occur in a crisis. If so, then its (original) more pessimistic CLO ratings could prove to more accurate than those of a Gaussian or unstressed BET model with corresponding parameter inputs (i.e. recovery rates, correlations and loan default probabilities).

In practice, ratings are not entirely based on models, so the risks identified in this section will be mitigated to an extent that ratings are informed by other evidence and assessments (from outside the model). In addition, the risks should be mitigated in part by the requirement that structured products have at least two ratings from different CRAs, because the credit assessment of a CLO tranche should then rely on at least two different models.

Conclusion

This paper has outlined some of the main approaches to modelling CLO credit risk by CRAs. Using simplified versions of the kinds of models used by CRAs, we simulated an idealised CLO to explore how tranche credit ratings could vary with different inputs, such as default correlation, and by the type of model used, such as BET or a particular copula. Thus, CLO ratings can vary as a result of small differences in parameter inputs, or when a different model is used.

The COVID-19 crisis is disrupting markets in ways that were impossible to predict when CLO ratings were assigned before the epidemic. It is likely that some CLO ratings will now evolve in response, with downgrades occurring as CRAs revise their views. With this in mind, model analyses like those presented here, but calibrated to the detailed CRA models, could help investors and other market participants understand where downgrades could eventually occur and how extensive they might be. This should help to moderate a possible procyclical impact of ratings from model risk, whereby ratings are subject to large downgrades, as a model's inaccuracies become clear in a stress period.

Indeed, the largest CRAs have recently published some scenario analyses looking at how the COVID-19 crisis might affect CLO ratings for their own models.[109] However, these look at how ratings would change under more detailed specific scenarios, and do not explain how their rating models would systematically change ratings as model inputs change, as attempted here for our simplified CLO.

More systematic and granular information on model inputs and the models themselves could help investors understand better some risks they implicitly take on by relying on ratings when investing in CLOs. For example, it could help to make explicit the extent to which an investment in a CLO tranche is a bet against recovery rates of the leveraged loans in the CLO pool turning out to be lower than originally anticipated. It could also help investors assess how cumulative changes in the different underlying model parameters might influence different CLO tranches.

Without systematic transparency on model risks and their potential impacts, for example, through sensitivity analyses or reverse stress tests, market participants are more likely to overlook some model risks and underprice them. In addition, as was seen in the GFC with CDO ratings and their implicit underpricing of correlation risk in US residential mortgage markets, this can have potentially detrimental impacts if the overlooked risk crystallises and downgrades follow.

This article also complements the findings of ESMA's recent thematic report,[110] which highlighted the importance of CRAs continuing to perform regular stress-testing simulations and to provide market participants with granular information on the sensitivity of CLO credit ratings to key economic variables affected by the COVID-19 pandemic.

References

AFME (2019), "European High Yield & Leveraged Loan Report European Leveraged Finance Q3 2019".

---

[109] See Moody's (2020) 'Shape of downturn, position in capital structure will influence collateral defaults', and S&P (2020) 'Scenario Analysis: How Credit Distress Owing to COVID-19 Could Affect U.S. CLO Ratings'.

[110] ESMA (2020).

Barclays Research (2020), "High yield, leveraged loans & CLOs: Supply update", 15 May 2020.

Bluhm, C., Overbeck, L., and Wagner, C. (2002), "An Introduction to Credit Risk Modelling", Chapman and Hall.

Cifuentes, A and G O'Connor (1996), "The Binomial Expansion Method Application to CBO/CLO Analysis", *Moody's Investors Service Structured Finance Special Report*, December.

ESMA (2019), "Leveraged loans, CLOs – trends and risks" in *ESMA Reports on Trends, Risks and Vulnerabilities*, No.2, 10 December 2019,

ESMA (2020), "Thematic Report on EU CLO credit ratings – an overview of Credit Rating Agencies practices and challenges". May.

FCIC (2011), "Final Report of the National Commission on the causes of the financial and economic crisis in the United States", Official Government Report, January.

Fender, I and J. Kiff (2004), "CDO rating methodology: Some thoughts on model risk and its implications", *BIS Working Papers*, No 163.

Fitch Ratings (2019), "CLOs and Corporate CDOs Rating Criteria", July.

FSB (2019), "Vulnerabilities associated with leveraged loans and collateralised loan obligations", December.

Moody's (2004), "Moody's Correlated BET Default Distribution".

Moody's (2009), "Moody's updates key assumptions for rating CLOs.", February.

Moody's (2018), "Moody's Idealized Cumulative Expected Default and Loss Rates", October."

Moody's (2019a), "Moody's CDOROM™ v2.17 User Guide".

Moody's (2019b), "Moody's Global Approach to Rating Collateralized Loan Obligations", March.

Natixis Asset Management (2017), "CLO 2.0: Mechanism, modelling and management", June.

Nickerson, J. and J.M. Griffin, (2017), "Debt correlations in the wake of the financial crisis: What are appropriate default correlations for structured products?", *Journal of Financial Economics*, p.454-74.

S&P (2019), "Global Methodology and Assumptions for CLOs and Corporate CDOs", June.

TwentyFourAM (2019), "CLOs: Europe vs US", 11 September 2019.