Product Intervention Analysis

Measure on Binary Options

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1 Executive summary

This document summarises and analyses relevant evidence in relation to the provision of binary options to retail clients. This document complements the *European Securities and Markets Authority Decision of 22 May 2018 to temporarily prohibit the marketing, distribution or sale of binary options to retail clients in the Union in accordance with Article 40 of Regulation (EU) No 600/2014 of the European Parliament and of the Council* (the Decision), providing additional detail. This document sets out:

(i)   analysis of the impact that different intervention options in relation to binary options would be expected to have on investors and on firms; and

(ii)   a study of the properties of the distribution of returns from investing in binary options.

In designing the product intervention measures through extended information-gathering and discussion with National Competent Authorities (NCAs), ESMA considered different alternatives. Section 2 of this document describes the different options considered and examines the expected impact of each option. Some impacts are described solely in qualitative terms, while in other cases quantitative information is available.

On 18 January 2018, ESMA issued a call for evidence on possible intervention measures, to gather additional information. While most respondents to the call for evidence focused on issues related to contracts for differences, a minority made reference to binary options. Section 2 of this document takes these views into account.

Two fundamental features of the return distribution of binary options are especially relevant from the perspective of possible intervention measures. These are (i) the high levels of risk involved in binary options, and (ii) the negative expected return of the product. Section 3 examines these features in some detail. A key consequence of the second feature is that the more often an individual invests in binary options, the more likely he or she is to suffer a loss on a cumulative basis, and the more likely he or she is to exhaust a given amount of money available for investing.

Binary options can be structured or provided in different ways. For example, some providers quote continual two-way pricing, allowing an investor to sell back an option before expiry. Section 3 sets out simulations based on some more standard products, using examples of specific numerical payoffs. Importantly, however, the analytical results apply in all cases, as demonstrated on theoretical grounds. Data presented in section 3 further corroborate the broad applicability of the analysis. Other data indicate the pertinence of the results relating to repeated trades.

2 Analysis of likely impact of different options

1. This section of the document:

   - describes what binary options are and describes the retail market for binary options;

   - explains the key features of the problem that has led ESMA to take action;

   - summarises the intervention measures that ESMA is introducing;

   - examines any potential impact the different intervention options could have on market efficiency and the wider financial system; and

   - analyses the expected direct impact of the different intervention options on investors and on firms.

2. More precise detail on what binary options are, the intervention measure and the problem addressed are in the Decision alongside which this document is published. [1]

2.1 Product description and overview of the retail market

2.1.1 Key product features

3. A binary option is a cash-settled derivative that expires at a pre-specified time. It generally has two possible outcomes at expiry: either it pays out a fixed monetary amount (the "fixed payoff"), specified in advance, or it is worth zero (i.e. the investor loses the entire investment). The option pays out at expiry if a specified event relating to the price of the underlying has occurred. If, at expiry, the specified event has occurred, the option is said to "finish in-the-money".

4. Commonly, the specified event that determines whether a binary option pays out is whether the price of the underlying at expiry exceeds the price of the underlying when the option was issued. A binary option based on a specified event of this sort is sometimes called an "up-down" binary option. Box 1 gives an example of an up-down binary option.

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[1] European Securities and Markets Authority Decision of 22 May 2018 to temporarily prohibit the marketing, distribution or sale of binary options to retail clients in the Union in accordance with Article 40 of Regulation (EU) No 600/2014 of the European Parliament and of the Council.

> **Box 1: example of an up-down binary option**
>
> As an example, consider a binary option that has as underlying the level of a large stock index. The binary option is issued at 9.30am on a particular day. If the level of the index at 4pm that day is higher than it was at 9.30am, the option pays out, i.e. it finishes in-the-money. In situations of this kind, the probability of finishing in-the-money is typically around 50%, because a stock index usually has an approximately even chance of rising over the course of a trading session.
>
> Suppose that the option costs €100 to purchase at 9.30am, and that it pays out €180 if it finishes in-the-money and zero otherwise. In this case, if the option finishes in-the-money, the investor makes a net profit of €80. If the option does not finish in-the-money, the investor's net return is a loss of €100, i.e. the entire value of the initial investment.
>
> Assuming the probability of finishing in-the-money is 50%, the mathematically expected value of the option is half of €180, i.e. €90, which is less than the purchase price.

2.1.2 Common characteristics of the market for products offered to retail clients

5. Binary options are often provided OTC to retail clients, and the investor generally has the provider as counterparty. Counterparty risk is not modelled here.

6. Binary options sold to retail clients usually have very short time horizons, in some cases as short as a few minutes.

7. Some providers offer continual two-way pricing. This means that retail clients can choose to sell back binary options they have purchased, before expiry. The amount the investor receives in return will be somewhere between zero and the fixed payoff. Alternatively, the investor can buy more of the same binary option. The provider is free to determine the buy price and sell price at any time, which are not specified in advance.

8. Continual two-way pricing is a feature of the way binary options are provided, as opposed to the binary options themselves. In other words, the binary option remains the same, contractually, whether or not two-way pricing is offered.

9. Because binary options offer 'all or nothing' payoffs, their return profile is not well-suited to be used for the purpose of hedging.

2.2 Summary of problem to be addressed by intervention measures

10. Retail clients have received poor outcomes from investing in binary options. Binary options provide investors with a negative expected return, with no appreciable offsetting benefits other than facilitating speculation. As a result, a large majority of retail client accounts lose money in aggregate (even though some investors may make profits, especially in the short term).

2.2.1 Geographical scope of problem

11. The issue has an EU-wide impact on retail client protection, as evidenced by investor outcomes in several Member States. In many cases, retail customers trade binary options with providers that operate cross-border.

2.2.2 Market failure analysis

12. The problem is due in part to informational asymmetries. A clear majority of retail clients lose money, indicating that retail clients as a group overestimate the net returns they are likely to receive and do not understand the intrinsic features of these products.

2.3 Summary of intervention measures

13. As set out in the Decision, ESMA is imposing a temporary (three-month) prohibition on the marketing distribution or sale of binary options to retail clients.

2.4 Systemic context and market efficiency

14. There is no evidence of systemic risk from prohibiting the provision of binary options to retail clients in the EU. The intervention is expected to have no discernible impact on prices or liquidity of underlying assets or of other derivative contracts. Many firms providing binary options to retail clients are believed not to generally hedge their exposure using options in institutional markets, and so there is no evident channel through which any material financial shock could be propagated to financial markets more broadly. Additionally, while ESMA does not have a conclusive estimate of the size of the retail market for binary options in the EU, data on the numbers of clients in different jurisdictions and average client losses suggest that the order of magnitude of volumes of retail binary options is small compared with institutional options markets. Data reported by NCAs referred to in the Decision suggest an approximate upper bound on numbers of accounts in the EU to be around 500,000. Assuming an average client loss of EUR 500 annually – which is of the order of average estimated losses estimated by NCAs – yields an approximate upper bound on industry net revenues to be EUR 250 million. If average net revenue for binary options providers is 2% of contract size (a conservative estimate, in the context of estimating an upper bound on total volumes) then total annual volumes in binary options held by retail clients in the EU are unlikely to exceed EUR 12.5 billion. This figure is a very small fraction of total amounts outstanding in institutional options markets, which exceed EUR 10 trillion according to data from the Bank of International Settlements.[2] Moreover, the total annual volumes of binary options traded by retail clients are likely to be far higher than the amount outstanding at any point in time, as the typical term of binary options is very short.

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[2] Source: Bank of International Settlements, data update 2 November 2017. Latest data update is available at https://www.bis.org/statistics/d5_1.pdf

2.5 Direct impact on firms and investors

15. The following part of section 2 deals with the costs and benefits that the relevant intervention options considered by ESMA in developing the temporary prohibition would be likely to have on investors and on firms. [3] The analysis includes both quantitative and qualitative evidence.

2.5.1 Baseline scenario

16. The baseline scenario against which the impacts of the intervention options are assessed is the regulatory regime to which the market is subject is in place prior to the commencement of the intervention measures in question. In particular, firms are assumed in the baseline scenario to be subject to the requirements of MiFID II / MiFIR, all other applicable EU Directives and Regulations and all national rules in force. For example, the baseline scenario includes the bans on commercialisation or advertising of complex products (including binary options) in place in Belgium and in France, as these policies are in place when the ESMA intervention comes into force.

2.5.2 Evidence base

17. In respect of the costs and benefits of the intervention measure to retail clients, the evidence base includes detailed quantitative analysis, whereas in respect of the costs and benefits to firms, the evidence is mostly qualitative.

18. Figures on client returns provided by NCAs are based on samples of client accounts as reported by firms. Pricing information used to simulate examples of expected returns for investors taken from firms' public websites and the exact numerical estimates (as opposed to the quantitative analysis of the structural properties of the return distribution) should be regarded as illustrative only.

2.5.3 Analysis of intervention options

19. In all cases, the options described below assume that binary options traded on a trading venue are in scope for the measures. Some respondents to the call for evidence argued that binary options traded on a trading venue, whether or not securitised, should be excluded from the scope of the measures. However, the features and characteristics of binary options, which are the fundamental source of the identified detriment to retail clients (and the subsequent significant investor protection concerns), remain the same whether or not these products are traded on a trading venue. In other words, binary options on a trading venue would still present a negative expected return to investors, while offering a payoff structure that is poorly suited to

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[3] Impacts on investors should be interpreted while bearing in mind the objective to limit the significant investor protection concern arising from the sale, distribution or marketing of binary options. In particular, it is important to note that aggregate monetary total costs and benefits, where obtainable, do not necessarily equate to aggregate detriment, e.g. in the following respects. (i) Some consumers may not expect certain costs, or understand the risk that the costs will materialise. If the costs do then materialise, the resulting detriment may be greater than if the consumers had anticipated the risk. (ii) The wealth and liquidity of consumers bearing costs also have a bearing on detriment. For example, a consumer whose losses exceed his or her cash balances with a firm and is unable to settle his or her account is likely to suffer considerably more detriment than an investor suffering a loss of the same size but which is covered by his or her cash balances. (iii) The distribution of costs on consumers is relevant to detriment. Other things equal, if a given aggregate loss falls on a smaller number of investors, this may cause more overall detriment than the same aggregate loss spread over a larger number of investors.

the purpose of hedging or other economic functions that could form a compensating benefit. Notably, these properties hold true at any time prior to the expiry of the option. The existence of any secondary market in binary options, therefore, does not change the fundamental characteristics that cause detriment to retail clients.

Intervention objective	Address the significant investor protection concerns arising from the sale, distribution or marketing of binary options.
Option A	Prohibit the marketing, distribution or sale of binary options to retail clients.
Option B	Restrict the provision of binary options to retail clients, e.g. impose limits on the amount of money that a client can invest per trade or per client, or impose a minimum term.
Option C	Prohibit the marketing, distribution or sale of binary options to all clients.
Preferred Option	Option A. As detailed below, there are common intrinsic features of binary options which are the fundamental source of detriment to retail clients. Therefore, limiting the range of binary options to which measures would apply (option B) is not warranted. On the other hand, the analysis conducted by NCAs and ESMA has focused on the significant investor protection concern arising from the detriment to retail clients and therefore option C is not selected at this stage.

Option A	Prohibit the marketing, distribution or sale of binary options to retail clients
Benefits	Retail clients would no longer suffer any monetary losses or other detriment in connection with binary options. They would be comprehensively protected from the high levels of risk arising from a structurally negative expected return product, a type of product which, in addition, is poorly suited to hedging and investment purposes.
Costs to regulators	The measure would need to be enforced and monitored. Monitoring costs would be lower than option B, however, since NCAs would not need to assess the features of binary options sold, distributed or marketed to retail clients.
Compliance costs	Firms would forego all future revenues arising from the provision of binary options to retail clients. Firms whose sole business is providing binary options could exit the market or would have to turn to other products (or to focus their offer to non-retail clients only). Firms who provide other products in addition to binary options would need to cease providing binary options, and as a result some such firms may exit the market.

	Some costs would be involved in updating IT systems. CMC Markets, in its response to the call for evidence, estimated the costs incurred by the firm in this respect to be around GBP 16,000. The cessation of losses by investors would approximately balance, in aggregate monetary terms, the lost revenue by firms.
Other costs	A minority of retail clients make positive net returns in the baseline scenario. Such positive net returns would not accrue to investors under options A or C. Some respondents to the call for evidence indicated that binary options on a trading venue should be excluded from the scope of the measures. However, the features and characteristics of binary options, which are the fundamental source of the identified detriment to retail clients, remain the same whether or not these products are traded on a trading venue, and any time prior to the expiry of the option. The existence of a secondary market, therefore, does not change the fundamental characteristics that cause detriment to retail clients, just as the possibility of two-way pricing offered by OTC providers does not alter such characteristics.

Option B	Restrict the provision of binary options to retail clients, e.g. impose limits on the amount of money that a client can invest per trade or per client, or impose a minimum term.
Benefits	Restrictions designed to limit the frequency (via a minimum term) or amount of investment by a client would aim at reducing the risk of substantial detriment. However, the analysis of the return distribution detailed in section 3 applies regardless of the initial investment. In particular, investors would still face a negative expected return, which would come in the absence of providing any compensatory benefits such as hedging. This would limit the benefit of the intervention option.
Costs to regulators	The measure would need to be enforced. Either a per-trade or per-account monetary limit or a minimum term requirement would require national authorities to monitor the detail of binary options offerings, which would incur higher costs for regulators than options A or C.
Compliance costs	Firms would incur one-off costs in changing their platform software and their terms and conditions and on-going costs to ensure ongoing compliance Some costs would be involved in updating IT systems, as for options A or C.
Other costs	A minority of retail clients make positive net returns in the baseline scenario. These investors would be constrained, but would still be able to trade binary options under option B, unlike under options A or C.

Option C	Prohibit the marketing, distribution or sale of binary options to all clients
Benefits	All clients would no longer suffer any monetary losses or other detriment in connection with binary options. They would be comprehensively protected from the high levels of risk arising from a structurally negative expected return product, a type of product which, in addition, is poorly suited to hedging or investment purposes. Extending the prohibition of marketing, distribution or sale of binary options to all investors would be warranted by the described features of the products and would consequently also benefit non-retail clients (professional clients and eligible counterparties). On the other hand, ESMA has no evidence that such clients actually use binary options. Additionally, non-retail clients would be less likely to invest in binary options without understanding the return profile and the pricing structure of these complex products, in contrast to retail clients, limiting the amount of any potential protection that a prohibition would provide such clients.
Costs to regulators	The measure would need to be enforced and monitored. Monitoring costs would be lower than under options A or B due to the complete ban of binary options for all categories of clients.
Compliance costs	Firms would forego all future revenue arising from the provision of binary options. Firms whose sole business is providing binary options would exit the market or would need to turn to other products. Firms who provide other products in addition to binary options would need to cease providing binary options, and as a result some such firms may exit the market. Some costs would be involved in updating IT systems, as for options A or B. The cessation of losses by investors would approximately balance, in aggregate monetary terms, the lost revenue by firms.
Other costs	A minority of retail clients make positive net returns in the baseline scenario. Such positive net returns would not accrue to investors under options A or C.

3 Quantitative analysis

20. This section of the document:

   - provides an overview of the features of the return distribution for retail clients in binary options;

   - states the main results mathematically; and

   - presents graphical simulation results and discusses relevant empirical data.

21. Section 3.1 discusses the main features of the return distribution. Section 3.2 then formally presents the main results. For clarity, graphs of simulation results accompany some of these results. The simulations use specific examples of payoff structures.

22. Section 3.3 demonstrates the general applicability of the results. In particular, it shows that the results are robust across different types of payoff structure – including where the strike price differs from the initial price of the underling – and that they hold whether or not the provider offers continual two-way pricing.

23. Section 3.4 provides relevant data and information from firms, and concludes that the available empirical evidence supports the (largely theoretical) modelling exercise.

24. A further, minor, robustness check is provided in Annex II, in relation to the assumed 50% win likelihood assumed when modelling up-down options. The robustness check is performed by using the Black-Scholes-Merton (BSM) pricing model. The reason that the check is of minor importance is that, as established in section 3.3, the 50% likelihood is not critical to the results. However, the check provides evidence that the specific modelling of up-down options reflects the actual return distribution for those options.[4]

3.1 Summary of main features of the return distribution

25. The theoretical distribution of returns from binary options has the two following fundamental properties.

   i. The high level of risk involved.

   ii. The negative expected return of the product.

26. The first feature is the high level of risk that the investor takes on in purchasing a binary option. A general feature of all binary options is that an investor risks losing his or her entire investment.[5]

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[4] This, in turn, corroborates the estimate for the negative expected return of around -10% for the up-down binary option modelled in section 3.2.2, which is based on a market provider's quote In other words, there is evidence that some binary options are sold at a large expected mark-up compared to fair value. However, the size of the mark-up as a potential source of investor detriment is not central to the case made for the intervention measure.

[5] The main quantitative analysis assumes that if option finishes out of the money it pays out zero. There is a small technical caveat to this assumption, which has no impact on the results. For more details, see the discussion in section 3.2.1.

As mentioned above, for a standard up-down binary option the probability of this happening is around 50%.

27. Even a single trade therefore involves a high level of risk. Risk alone is not necessarily a source of investor detriment, if an investor takes on risk because he or she reasonably expects higher returns, for example. However, binary options do not compensate investors for the risk they take on by offering commensurate expected rewards. For this reason, the level of risk represents a source of investor detriment (and therefore a significant investor protection concern).

28. The second feature, the negative expected return of a binary option for the investor, refers to the mathematical expectation of the return on the investment. It means that if the option's fixed payoff is multiplied by the probability of finishing in the money, the result is less than the initial amount invested. Taking the example in Box 1 in section 2.1.1, the product has a 50% chance of finishing in-the-money, and if it does so, the investor receives €180. The expected net value of the binary option is therefore half of €180, i.e. €90. The initial investment of €100 is greater than the expected net value of the option, which means that the investor's expected return is negative.

29. The negative expected return does not necessarily require that the investor is more likely to lose than to win. In many cases, such as for up-down binary options, the probability of winning is around 50%. But in other cases, the probability may be different. Nonetheless, the expected return will be negative. Investors must face negative expected returns, or providers' business model would not be profitable

30. In the context of binary options, the negative expected return is another important source of detriment and applies to all binary options. While the existence of a negative expected return on an asset or financial instrument is not in itself grounds for concern (for example, an insurance contract is valuable to a risk-averse investor despite having a negative expected return), binary options do not provide any compensating benefit in terms of economic functions such as insurance, hedging or investment. Unlike financial investments, the contracts are typically very short term and do not offer participation in the growth in value of the underlying. Unlike vanilla options, which are often used for hedging purposes, binary options provide a fixed payoff if a specified event occurs. In contrast, the payoff of a typical option is contingent on the change in the price of the underlying once the option is in the money (i.e. the payoff is variable). This inherent feature of the products limits the value of the product as a hedging tool, whereas other kinds of option have been used to smooth out or limit the price of an asset to which a firm or an investor has direct exposure.

31. An important consequence of the fact the investor faces a negative expected return is that the more often he or she invests, i.e. the more binary options he or she bets on, the greater the probability of suffering a loss on a cumulative basis. This fact ties in to a simple interpretation of the expected value of a gamble as being the long-run average outcome of many repeated trials.

32. Relatedly, repeated investment (i.e. placing a series of bets) also increases the probability that an investor entirely depletes a given amount of money available for investing.

3.2 Main results

3.2.1 Notation

33. In stating the payoffs of an option, it is simplest to normalise by assuming an investment of 100 units of currency. Let the *payoff vector* (a, b) denote the payoffs for an investor, where:

   - the first entry, a, denotes the gross payoff if the option does not finish in-the-money; and

   - the second entry, b, denotes the gross payoff if the option finishes in-the-money

34. For example, the payoffs of the binary option in Box 1, which pays out €180 (gross) on a €100 investment, and zero otherwise, are denoted $(0, +180)$.

35. This notation denotes only the payoffs, and not the probability distribution over the payoffs. This means that simply comparing two binary options' respective payoff vectors does not reveal which has the higher expected value, unless the probability of finishing in-the-money is also known. For example, if two options have identical payoff vectors, but one is 55% likely to finish in-the-money while the other only has 45% likelihood, the former has a higher expected value.

36. For generality, the notation allows for the possibility that $a \neq 0$. Some providers offer options that pay a small amount if the option does not finish in-the-money. This offering is equivalent to the investor making a small payment to the provider that is due to be returned unconditionally, alongside purchasing an option that pays zero if it does not finish in-the-money.[6] While it is convenient to assume that $a = 0$ for the theoretical results in this analysis, the results in fact hold more generally.

37. Other notation used is as follows. Define p to be the probability of finishing in the money, so that the expected value of payoff vector (a, b) is given by $(1 - p)a + pb = pb$ (recalling that $a = 0$). The expected return $r(a, b, p)$ – expressed as a proportion of the initial investment of 100 – is then as follows: [7]

$$r(a, b, p) = \frac{(1 - p)a + pb - 100}{100} = \frac{pb}{100} - 1$$

38. For clarity, the notation hereafter omits the arguments when denoting the expected return, i.e. simply writes it as r.

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[6] Abstracting away from counterparty risk, this can be seen mathematically as follows. Consider a binary option with payoff vector $(+5, +175)$. The payoff vector can be written $(+5, +175) = (0, +170) + (+5, +5)$, i.e. a binary option with a gross payout of 170 and zero otherwise, plus a certain return of 5.

[7] The normalisation of assuming an investment of 100 makes no difference to the results. One way to see this is that the definition of r can simply be made scale-invariant by allowing for the initial investment to be a variable y. In this case, if (a, b) is the payoff vector for an investment of 100, then $\left(\frac{y}{100}a, \frac{y}{100}b\right) = \frac{y}{100}(a, b)$ is the payoff vector from investing y. The return is then given by $r = \frac{\frac{y}{100}[(1-p)a+pb]-y}{y} = \frac{(1-p)a+pb-100}{100}$, as before.

3.2.2 Assumptions

39. The results that follow are based on the following key assumptions:

- *Assumption 1: $r < 0$*. This assumption is justified by the simple fact that offering binary options must yield expected profit for the provider, or the business model used by providers would not be sustainable.

- Assumption 2: in any series of investments in binary options, the outcome of each investment (i.e. whether it finishes in-the-money) is realised independently of the outcomes of the other investments.

3.2.3 General theoretical results

40. The long-run effects of repeated investments in any binary option (for which $r < 0$) are summarised in the following theoretical results.

   a) If an investor makes a large number of investments, then the investor is unlikely to make a positive overall return. Formally, the probability that the sum of the investor's returns is positive will tend to zero as n tends to infinity.

   b) If an investor has a limited amount of cash available to invest in binary options, and makes a large number of investments, then the investor is likely to lose all the cash. Formally, this result is implied by the fact that the probability that the sum of the returns is less than any given finite number will tend to one as n tends to infinity.[8]

41. These results are proved in Annex I. Chart 1 illustrates the properties graphically, for the example payoff vector $(0, +180)$. The horizontal axis denotes the number of trades and the horizontal axis represents the probability mass density of the distribution. It can be seen from the chart that the expected return (mean) of the distribution linearly decreases in the number of trades, as $-nr$. The proportion of each curve lying below a given constant – for example, zero on the horizontal axis – is also seen to decrease as the number of investments n increases.

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[8] The reason the formal statement is in fact a sufficient (but not a necessary) condition for the first sentence in result (b) is as follows. An available cash sum is represented as some finite number. The formal statement says that a large series of returns will converge in probability to less than that number, i.e. the cash will be depleted in the long run. However, it is also possible that a short series of returns may, through chance, deplete the entire cash sum. As the cash sum is a liquidity constraint, trading would then cease with the cash sum exhausted.

Chart 1: distribution of aggregate return from selected numbers of repeated investments of 100 in binary options with payoff vector $(0, +180)$ when $p = 0.5$ and investment outcomes are realised independently
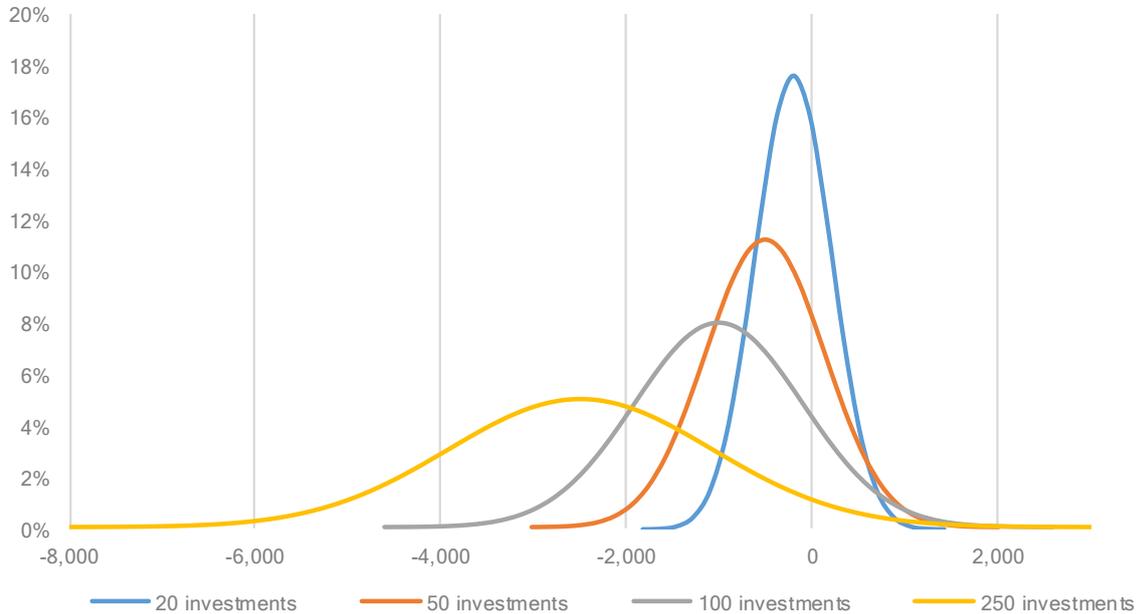
Note: Probability mass density, according to binomial distribution, of aggregate net return for selected total numbers of investments made, where each investment is of size 100 and the payoff vector is $(0, +180)$. Each investment is assumed to finish in-the-money with a probability of 50% and all outcomes are assumed to be realised independently. Plotted distributions are truncated for clarity (i) for horizontal axis values below -8000 and above +3000, and (ii) for vertical axis values below 0.001%.
Sources: AFM, ESMA.

3.2.4 Numerical analysis

42. The theoretical results are very general, applying to all binary options (assuming negative expected returns). However, by the same token, they are also abstract. For concreteness a specific worked example is first presented, before turning to graphical illustrations for different numerical payoff vectors and win probabilities.

3.2.4.1 Worked example

43. Consider the payoff vector of Box 1, $(0, +180)$. As the present focus is an up-down option, set $p = 0.5$. As noted in Box 1, $r < \frac{90 - 100}{100} = -10\% < 0$.

44. The expected return after making $n \geq 0$ such investments is simply $r \times n = -0.1n < 0$.

45. The distribution of returns after making $n \geq 0$ such investments is given by the binomial distribution. Specifically, the probability that the investor wins exactly k times from n investments is given by

$$Pr(k; n, p) = \binom{n}{k} p^k (1 - p)^{n-k}$$

where $\binom{n}{k} \equiv \frac{n!}{(n-k)!k!}$.

46. As $p = 0.5$ in this case, the expression simplifies to

$$Pr(k; n, 0.5) = \binom{n}{k} 0.5^n$$

47. Let the function $q(n) > 0$ denote the minimum number of wins an investor needs from $n > 0$ investments such that the aggregate return is positive. In other words, for given n, $q(n)$ is the smallest integer such that the following inequality holds.

$$q(n)b > 100n$$

48. Suppose a customer makes 20 investments, so that $n = 20$. The overall number of wins is randomly determined, and must lie between 0 and 20. To work out the investor's probability of winning overall, the first step is to work out the minimum number of wins out of 20 investments consistent with an overall win, when the amounts won and lost are added up. In other words, it will be necessary to determine $q(20)$.

49. It turns out that $q(20) = 12$. This can be seen by first considering what happens when the overall number of wins happens to be 11, which yields a net cumulative return of $9 \times \left(\frac{a-100}{100}\right) + 11 \times \left(\frac{b-100}{100}\right) = 9 \times (-1) + 11 \times 0.8 = -0.2 < 0$.

50. In other words, if an investor make 20 investments, each of €100, in the binary option in question, then if there are 11 wins and 9 losses, the investor loses 20% of the initial stake, or €20.

51. If the overall number of wins happens to be 12, the net cumulative return is €160. This is because the net cumulative return, as a proportion of a €100 investment, is $8 \times (-1) + 12 \times 0.8 = +1.6 > 0$.

52. The binomial distribution then implies that the probability of winning on a cumulative basis after 20 investments is given by

$$Prob(overall\ win) = \sum_{k=12}^{20} \binom{20}{k} 0.5^{20}$$

which is around 25%. This implies that the probability of *losing* overall is around 75**%**.

53. A similar procedure yields a lower bound for the probability of depleting a given amount of cash available for investment.[9] For instance, suppose the investor places 20 trades each of €100. If the number of wins happens to be 6, then the investor loses €920, as

$$14 \times (-1) + 6 \times 0.8 = -9.2$$

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[9] The reason that the procedure determine a lower bound for the probability of depleting available cash, rather than the actual probability, is that it assumes the investor will make all 20 investments. It excludes, for instance, sequences in which that the investor's first 10 investments all lose, but the total number of subsequent wins is greater than 5. As by hypothesis €1000 is the total available budget for investment, such sequences represent cases where the investor's entire budget would be depleted. For present purposes, however, the lower bound is instructive, as the aim is simply to illustrate general properties through specific numerical examples.

54. If the number of wins happens to be 5, then the overall return is a loss of €1100, as

$$15 \times (-1) + 5 \times 0.8 = -11$$

Hence the probability of entirely depleting a €1000 budget for investment satisfies the following inequality.

$$Prob(exhaust \; €1000 \; budget\,) > \sum_{k=1}^{5} \binom{20}{k} 0.5^{20}$$

i.e. the probability of exhausting the budget is greater than 2%.

3.2.4.2 Graphical simulation results

55. Using the procedure outlined in the worked example above, it is possible to simulate overall loss probabilities for any given payoff vector, for different numbers of trades. Chart 2 presents overall loss probabilities for sequences of €100 investments in binary options with payoff vector $(0\,, +180)$ and win probability $p = 0.5$, where investment outcomes are realised independently. The length of the sequence n varies on the horizontal axis. The loss probabilities plotted are specifically (i) the probability of overall loss, after n investments; and (ii) the probability of losing at least €1000, €2000 and €3000 respectively.[10]

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[10] As each investment is of size 100, a sequence length of $n = 50$, for instance, represents a total gross investment of 5000.

Chart 2: simulated probabilities of cumulative loss exceeding given amounts for repeated investments of 100 in binary options with payoff vector $(0, +180)$ when $p = 0.5$ and investment outcomes are realised independently
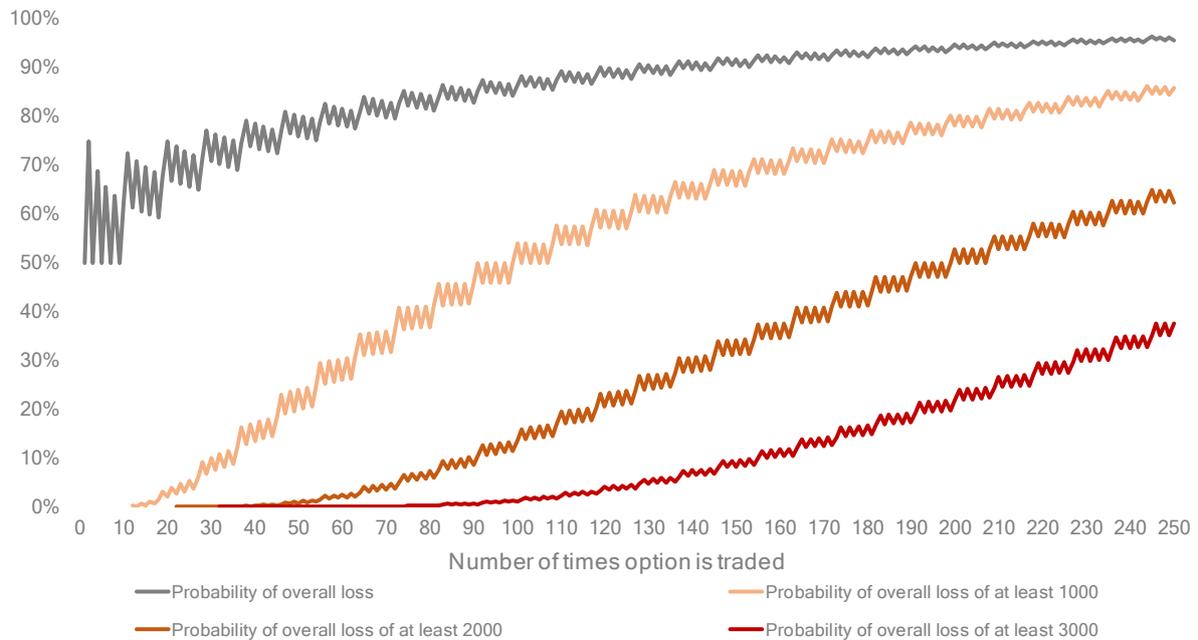
Note: Loss probabilities are those of an investor with unconstrained liquidity. For a liquidity-constrained investor, simulated values are lower bounds. Binary option simulated has gross payoffs of 180 when finishing in-the-money and zero otherwise, on an investment of 100. Each trade is assumed to finish in-the-money with probability of 50%, with all outcomes realised independently. Probabilities are plotted for integer values on horizontal axis, with lines joining these points for clarity.
Sources: AFM, ESMA

56. While the actual probabilities plotted in Chart 2 are only for integer values of trade numbers, the results are plotted with lines in between these points for visual clarity. To interpret the graphical results, note first that for a single trade, an overall loss requires that the investor lose that single trade; the overall loss probability therefore starts equal to p for $n = 1$. For an up-down option, a negative expected return implies that one win and one loss from two investments results in an overall loss, and so the only way to win overall for $n = 2$ is for both investments to pay out. This is why the overall loss probability jumps to 75% for $n = 2$. For the first few odd values of n, there needs to be one more loss than wins for an overall loss, whereas for even values of n, there need to be two more, which explains the "zig-zag" pattern. Periodically, the proportion of losses needed for an overall loss reduces by one for both odd-length and even-length sequences, which explains the less frequent, larger step increases in loss probability as n increases. Finally, note that all the plots form a family of "S"-shaped curves (most clearly seen in the case of probability of overall loss of at least 1000, given the scale on the horizontal axis), whose values will converge on 100% for large n.

3.3 General applicability

57. The analysis of section 3.2 focused on standard up-down options. For up-down binary options, the specified event on which payoffs are conditioned (the "trigger event") is that the price of the underlying at expiry exceeds the price when the binary option was issued.[11] This section shows that the properties of the return distribution identified in section 3.2 hold for all binary options.

3.3.1 Different trigger events

58. Some providers offer binary options with different kinds of trigger event. The use of terminology in this context can vary between firms. Beyond the standard up-down binary option, other types of trigger event include, but are not limited to, the following.[12]

   - The option finishes in the money if the price of the underlying at expiry exceeds the strike price, where the strike price differs from the initial price. Some providers call this a "ladder" option.

   - The option finishes in-the-money if the price of the underlying exceeds the strike price *at any time* before expiry.[13] Some providers call this a "touch" option.

   - The option finishes in-the-money if the price of the underlying is within a certain range at expiry. The payoff of a target option is therefore mathematically equivalent to that of two up-down binary options, a short option with a strike price equal to the higher end of the range and a long option with a strike price equal to the lower end of the range.[14] Some providers call this a "range" option.

59. Section 3.2 focused on modelling up-down binary options, as these are routinely offered by many binary options providers and provide a clear reference point. Larger providers tend to offer other types of binary option alongside up-down binary options.

60. Because the underlying event can vary, the value of the other types of binary options may be especially hard for retail clients to understand. For instance, a "touch" option is more likely to finish in-the-money than an up-down option with the same underlying, strike price and expiry time. The reason is intuitive: for the touch option to pay out, the price of the underlying simply needs *at some point* during the term to "touch" (or cross) the relevant threshold, rather than to be at or above the threshold at expiry. However, estimating exactly how much more likely the

---

[11] This description assumes a long option. If the option is short rather than long, the option is in-the-money if the price of the underlying is less than the initial price.

[12] Some offerings may involve a series of trigger events, with more than one fixed payoff depending on which events in the series occur. Such offerings are in effect a sequence of dual-payoff binary options.

[13] Unlike the other types of product listed, such an option would therefore probably be priced as an American option, rather than as a European option.

[14] For example, suppose such an option has a gross payoff of +50 if and only if the underlying price ends between €35 and €36, and 0 otherwise. This has the same payoff as the following pair of options: (i) a long binary option that has a gross payoff of +25 if the underlying price ends above €35 and -25 otherwise; (ii) a short binary option with a gross payoff of +25 if the underlying ends below €36 and -25 otherwise. Taking the payoffs of the pair together, the investor receives +25-25=0 if the price of the underlying ends below €35, 25+25=50 between €35 and €36, and -25+25=0 if the price of the underlying ends above €36.

touch option is to pay out than its up-down equivalent is complicated, because it depends not just on where the price ends up but what path it takes before expiry.

61. For the purposes of this analysis, attention is focused on the fundamental properties of the return distribution of any given binary option – namely the high level of risk and the negative expected return – and the effects of repeated trades. Importantly, these implications hold generally. The reason is that in all cases, a binary option pays out a fixed monetary amount with some given ex-ante probability.

62. To illustrate the fact the results extend beyond up-down options to all binary options regardless of the type of trigger event, Chart 3 models a binary option with a payoff vector of $(0, +111.7)$ and a win probability of $p = 87.85\%$.[15] These parameters are taken from an example of provider pricing information, where the win probability is inferred as the mid-point of a buy and sell price, leading to an assumed value of p quoted with high precision.

Chart 3: simulated probabilities of cumulative loss exceeding given amounts for repeated investments of 100 in binary options with payoff vector $(0, +111.7)$ when $p = 87.85\%$ and investment outcomes are realised independently
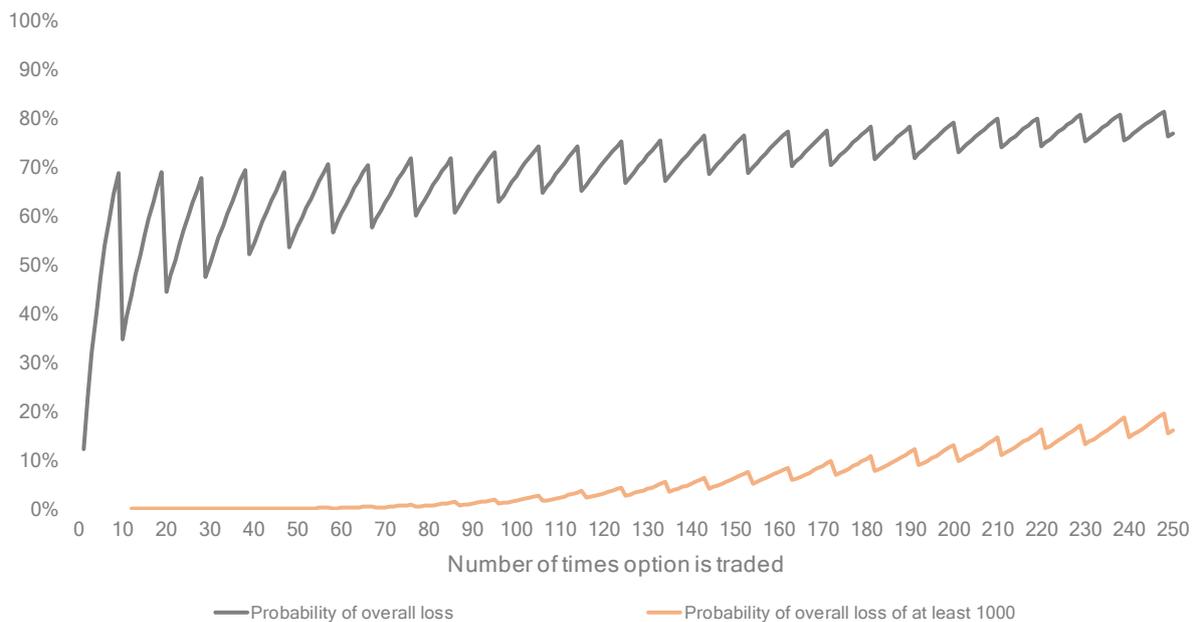
Note: Loss probabilities are those of an investor with unconstrained liquidity. For a liquidity-constrained investor, simulated values are lower bounds. Binary option simulated has gross payoffs of 111.7 when finishing in-the-money and zero otherwise, on an investment of 100. Each trade is assumed to finish in-the-money with probability of 87.85%, with all outcomes realised independently. Probabilities are plotted for integer values on horizonal axis, with lines joining these points for clarity.
Sources: AFM, ESMA

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[15] In Chart 3, attention is restricted to the probability of overall loss and the probability the investor loses at least 1000, in contrast to Chart 2, where probabilities of losing at least 2000 and at least 3000 respectively are also presented. The reason is that while the latter two probabilities also increase over time for the choice of parameters in Chart 3, they do not increase substantially over the number of trials modelled (from $n = 1$ to $n = 250$), so are omitted for clarity of illustration.

63. The binary option modelled in Chart 3 assumes a high win probability; in around 7 out of 8 trades, the option pays out. In addition, the assumed payoff structure and win probability are based on information from a large firm able to offer relatively competitive pricing, and so the expected return, while negative, is higher than that of the binary option of Chart 2. This explains why the results in Chart 3 show lower cumulative loss probabilities in general than in Chart 2. However, as Chart 3 shows, even in the case of a payoff structure of this kind, binary options present an inherent risk of substantial investor detriment, especially from repeated trading, which the available data (discussed in section 3.4) suggest is common practice among many retail clients.

3.3.2 Continual two-way pricing

64. In addition to the possibility of different types of trigger event, binary options may be provided in different ways. Specifically, as noted in section 2, regardless of the trigger event there may be a facility for the investor to "cash in" the option, i.e. sell it back to the provider having previously bought it (or buy it back from the provider having previously sold it). As the example of continual two-way pricing of an up-down binary option in Box 2 makes clear, two-way pricing involves a spread. This spread is in fact a spread in expected value of the option itself rather than a spread in underlying price. As such, it is probabilistic in nature: just as with the initial purchase of any binary option, in mathematical expectation the investor suffers a loss due to the spread. This is because the investor can sell an option to the provider for less than its expected value, or buy one for more than the expected value.

> **Box 2: example of two-way pricing of an up-down binary option**
>
> An up-down binary option which costs €100 to purchase has a payoff vector of $(0 , 190)$, and the time to expiry when it is issued is 3 hours. 1 hour after purchase, the price of the underlying has fallen, so the probability of finishing in-the money is lower. Suppose that at this time, $p = 25\%$. In this case, the expected value of the option is $25\% \times €190 = €47.50$.
>
> The provider continually quotes a price to the holder of the binary option to "cash out" the position. 1 hour after purchase, the firm quotes a price of €45 for the investor to sell the option back to the firm. The firm also quotes a price of €50 for the investor to buy another unit of the binary option.

65. The rest of section 3.3.2 examines the possible effects of the existence of two-way pricing on the ex-ante return distribution of a binary option

66. Two binary options are defined in this context to be *equivalent* if they have the same time to expiry, strike price, initial price and payoff vector. If early buy-backs are assumed to be at fair value (with zero fees), a binary option without two-way pricing and its equivalent with two-way pricing have same expected return.[16] This is because at any time t in between the start and the expiry of a binary option, the expected price $p(t)$ should be equal to the expected price (taken

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[16] More precisely, the same time-discounted expected return. Binary options are typically issued with a short time-to-expiry and time discounting is therefore negligible.

at t) that the option will have at any $t' > t$, neglecting discounting, as stated in the following equation.[17]

$$price(t) = e^{r(t-t')}E_t[price(t')]$$

67. In reality, the provider will make expected profit from early buy-back, for instance pricing using a spread of the kind illustrated in Box 2. In this case, a binary option clearly has no greater *ex ante* expected return if it is provided with continual two-way pricing than if it is not.

68. Further comparison of the risk involved in a binary option without two-way pricing versus its equivalent with two-way pricing is hindered by the fact that a determinate risk profile for the latter requires assuming a decision rule for the investor as to when to cash out. This assumption in turn would be stylised at best, taking into account the body of evidence in behavioural economics on, for instance, loss aversion. Rather than impose such an assumption, it is instructive to infer the following.

   - If an investor cashes out a binary option before expiry with non-zero probability, then the availability of two-way pricing reduces the investor's chance of losing the entire investment. (It also reduces the chance of receiving the maximum possible return.)

69. This result follows simply from observing that if an investor cashes out a binary option with non-zero probability before expiry, this must yield a return greater than zero and less than the in-the-money payoff at expiry. Had the investor not cashed out in such a situation, he or she would receive either zero or the maximum payoff at expiry. Consequently, the ex-ante probability of receiving zero or the maximum payoff from a binary option provided with continual two-way pricing is lower than if the investor can never cash out early.

70. Beyond this observation with regard to maximum and minimum possible returns, it is not possible to establish that a binary option is more or less risky due to continual two-way pricing, without imposing assumptions on investor behaviour – i.e. under precisely what conditions investors will and will not cash out a binary option.[18] As regards expected return, however, when investors cash out they cross a spread or incur a fee to do so, and so the expected return is reduced in practice by the availability of two-way pricing.

3.4 Empirical evidence

71. This section sets out statistics based on firm-level data on aggregate client outcomes to complement the theoretical analysis undertaken. In general, where the results can be 'sense checked' against the data, the data accord closely with the results.

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[17] The equation can intuitively be established by an arbitrage argument: if the buyer (assumed to be risk-neutral in the BSM model) knew that an asset were expected to appreciate in value above the risk-free rate of return, then he or she would be strictly better off waiting to buy the asset.

[18] An analytical tool used in the theoretical economics literature to compare the riskiness of distributions is second-order stochastic dominance. One gamble second-order stochastically dominates another if and only if all risk-averse expected-utility maximisers prefer the first to the second. Equivalently, this means the second gamble is a mean-preserving spread of the first. The ex-ante return distribution of an option provided with two-way pricing will depend on exactly under what conditions the investor cashes out. The precise assumption of investor behaviour therefore determines whether an option with two-way pricing second order stochastically dominates the same option without such pricing, or vice versa,

72. Statistics are as provided by NCAs and reported by individual firms. ESMA is not responsible for the accuracy of the data.

3.4.1 Firm-level data on aggregate client outcomes

73. Although ESMA does not have a detailed breakdown of data of the underlying assets, terms and the strike prices used by firms, aggregate data is available indicating outcomes for retail clients during 2015 and 2016 for 3 firms. (At least two of the firms, A and B, are known to be large providers.) In interpreting the summary of these results in Table 1, it is important to note that the data are self-certified by the firms and are not subject to independent scrutiny. Only firm B gave a breakdown in terms of up-down vs "volatility" binary options. Notably, "volatility" binary options in this context may include products other than those with up-down payoffs (cf. the product descriptions involving trigger events, in section 3.3.1), and/or products offered with continual two-way pricing.

74. Firm A referred to the use of a spread in written documentation accompanying its data submission, implying that the average expected value of an option when issued was 83% of the amount invested. It noted further that this figure might be lowered in future, as since launching the product the firm had been aiming to win market share.

75. As noted above, the analysis herein neglects explicit transaction fees, but the results naturally extend to business models that depend on transaction fees either instead of, or in addition to, a spread. Of the three firms, firm C appears to use transaction fees, whereas firms A and B rely on a spread. In all cases, as set out in Table 1 and as assumed in the analysis with respect to expected returns, clients on average lost money (see final column of Table 1).

Table 1: win and loss frequencies for retail investors per trade and per account				
	Proportion of trades in which investor won	Proportion of trades in which investor lost	Proportion of accounts with positive cumulative return	Proportion of accounts with negative cumulative return
Firm A (all products)	54%	46%	17%	83%
Firm B (up-down binary options)	50.7%	49.3%	16.7%	83.3%
Firm B ("volatility" binary options)	44.9%	55.1%	26.3%	73.7%
Firm C (all products)	49% (excluding fees)	51% (excluding fees)	15%	85%

Note: Statistics as provided by NCAs and reported by individual firms. ESMA is not responsible for the accuracy of the data. Statistics on accounts exclude zero-return accounts. Figures are inclusive of any fees unless otherwise stated. Firm A reported that statistics were for all binary options, based on trades from July 2015 to December 2016. For Firm B, the period was 2016 and statistics were split by up-down versus "volatility" binary options. For Firm C, all binary options are in scope of the statistics and the period was June 2015 to March 2017.

76. The specified value of a win probability of 50% when simulating up-down options, as done in the exercise illustrated by Chart 1, is borne out in all cases in Table 1. Table 1 shows that at an account level, a clear majority of accounts are loss-making for clients. This finding corroborates the key assumption that binary options are negative expected return products for clients for their sale, as implied by firms' business model. Table 1 is also consistent with the key result that due to the fact many clients engage in repeated trades, their probability of aggregate loss is significantly higher (between 73.7% and 85% according to the data on actual investor outcomes in Table 1) than that for a single trade (of between 44.9% and 54% according to Table 1).[19]

77. For the firm providing a breakdown of "volatility" binary options versus up-down binary options, it is evident from Table 1 that trades in volatility binary options were more frequently loss-making for investors, but that at an account level (i.e. looking at cumulative returns of all an investor's trades), volatility products resulted in losses less frequently. This pattern is hard to interpret in the context of the theoretical results of section 3.3, as the typical features of the binary options in question were not disclosed. Additionally, it is possible that "volatility" binary options were traded less frequently by many investors than standard up-down binary options, which would explain why fewer accounts trading "volatility" binary options realised an aggregate loss.

78. The main conclusion regarding the breakdown of data for up-down and "volatility" products is therefore that the results do not differ radically between the two types of binary options, supporting the analysis of section 3.3.

3.4.2   Evidence on repeated trades

79. One of the firms in Table 1 provided statistics on the total number of trades and the total number of unique clients. For the most recent year reported, 2016, these figures implied an average number of trades per client of between 100 and 200. Of course, some customers are likely to trade much more frequently and others much less frequently than this. The figure is useful in highlighting that many clients engage in repeated trades, however. This finding illustrates both how the products can encourage compulsive behaviour and, more relevant to the present analysis that restricts attention to return distributions, the importance of considering the effect of long-run repeated investing.

80. Another available data source comprises statistics reported via two other firms, as set out in Table 2. The monthly averages in the table indicate the existence of repeated trading.

Table 2: monthly average number of transactions reported per account			
	July 2017	August 2017	September 2017
Firm D	29	33	27
Firm E	50	54	44
Note: Statistics as provided by NCAs and reported by individual firms. ESMA is not responsible for the accuracy of the data.			

[19] These figures also accord with results provided by an NCA based on data from 10 binary options providers from January to August 2017, where it was found that on average, 87% of client accounts made a loss over the particular period.

4 Annexes

4.1 Annex I – Proof of general results

81. There are two general theoretical results:

   (a) The probability that the sum of the investor's returns is positive will tend to zero as n tends to infinity.

   (b) The probability that the sum of the investor's returns is less than any fixed number will tend to one as n tends to infinity. (This is a sufficient condition for the investor's cash balance being depleted in the long run.)

82. Both these results are established by considering the sample average

$$S_n := \frac{1}{n}(X_1, \dots, X_N)$$

83. where X_i is the ith observation of an investment return. S_n is thus a random variable that represents the investor's average return after a sequence of $n \geq 1$ investments, while nS_n is the investor's aggregate return.

84. As $n \to \infty$, the weak law of large numbers implies that S_n converges in probability to the population mean, i.e. to the expected return of an investment $r < 0$. Convergence in probability means that for any $\varepsilon > 0$, $Prob(|S_n - r| > \varepsilon)$ tends to zero as $n \to \infty$.

85. Result (b) says that for any constant c, $Prob(nS_n > c)$ tends to zero as $n \to \infty$, i.e. that $Prob(S_n > \frac{c}{n})$ tends to zero as $n \to \infty$. As this includes the case $c = 0$, (b) implies (a), so it is sufficient to prove (b).

86. Setting $\varepsilon = -\frac{r}{2} > 0$ shows that $Prob(|S_n - r| > -\frac{r}{2})$ tends to zero as $n \to \infty$, which implies $Prob(S_n > \frac{r}{2})$ tends to zero as $n \to \infty$. Consider sufficiently large n' such that $\frac{c}{n'} > \frac{r}{2}$. Then for all $n \geq n'$, $Prob\left(S_n > \frac{r}{2}\right) > Prob(S_n > \frac{c}{n})$. Hence as $n \to \infty$, $Prob(S_n > \frac{c}{n})$ tends to zero, as required.

4.2 Annex II – Application of BSM pricing to up-down options

87. This Annex shows that for a range of plausible parameter values for up-down binary options, the win probability according to the BSM pricing model is well-approximated by setting $p = 50\%$.

88. BSM options pricing is based on a theoretical assumption that the price of the underlying asset follows geometric Brownian motion with constant drift μ and standard deviation σ.

89. Let S_0 denote the initial price of the underlying, let K denote the strike price and let the random variable S_T denote the price on the expiry date T.[20]

90. A binary call option with a strike price K and expiry date T pays out an amount $a \geq 0$ if $S_T \leq K$ and pays out $b > a$ otherwise.[21] If $S_T > K$ then the option is said to be 'in the money'. Formally, the payoff at expiry is:

$$payoff = \begin{cases} a, & S_T \leq K \\ b, & S_T > K \end{cases}$$

91. According to the BSM theory, $\ln S_T$ is normally distributed with

$$E[\ln S_T] = \ln S_0 + (\mu - \frac{1}{2}\sigma^2)\, T$$

and

$$\text{Var}[\ln S_T] = \sigma^2 T$$

Thus

$$\frac{\ln S_T - \ln S_0 - (\mu - \frac{1}{2}\sigma^2)\, T}{\sigma\sqrt{T}}$$

is a standard normal variable with expected value of zero and standard deviation of one. This fact will help in calculating the probability that the binary option finishes in the money.

92. The binary option finishes in the money if $S_T > K$ or equivalently if $\ln S_T > \ln K$. The probability of this happening is given by $1 - N(x^*)$ where

$$x^* = \frac{\ln K - \ln S_0 - (\mu - \frac{1}{2}\sigma^2)\, T}{\sigma\sqrt{T}}$$

and $N(.)$ is the cumulative standard normal distribution function.

---

[20] More generally, T denotes the term of the product. As the term is to be measured in trading days in much of the analysis, 'expiry date' describes T, though more generally it is a time variable.

[21] This is the same payoff notation used in section 3.

93. Options can be valued in the BSM framework using risk-neutral pricing, i.e. assuming that all assets earn the same rate of return, $\tilde{r}$, as the riskless asset. Thus μ can be replaced by $\tilde{r}$ in the above equation to get

$$x = \frac{\ln K - \ln S_0 - (\tilde{r} - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

94. Note that the term is to be measured in trading days, and so r is the daily risk free rate and σ the daily price volatility. In practice, the volatility affects the results much more than the risk free rate.

95. The probability the binary option finishes in the money is then estimated by $1 - N(x) = N(-x)$. The option pays out only after T periods. Thus the expected discounted value of the binary option is

$$price = e^{-\tilde{r}T}(aN(x) + bN(-x))$$

96. The remainder of the analysis imposes parameter values designed to simulate approximate upper and lower bounds on the probability of finishing in-the-money implied by a binary option's BSM price, assuming the initial price equals the strike price, i.e. $K = S_0$. For the payoffs of the binary option it is simply assumed, without loss of generality, that $b > a > 0$. The strategy adopted here for simulating approximate upper bounds is to use values that are "extreme" in the context of the range of products offered to retail customers and to exploit the known properties of the BSM formula.

97. It can be verified by taking partial derivatives of the relevant algebraic expressions that the probability of finishing "in the money", $p = 1 - N(x)$, is:

- increasing in both the time period of the contract T and the volatility σ if $(\tilde{r} - \frac{1}{2}\sigma^2) > 0$, decreasing if $\left(\tilde{r} - \frac{1}{2}\sigma^2\right) < 0$ and independent of time period otherwise; and

- increasing in the rate of return $\tilde{r}$ whatever the values of the other parameters.

98. Correspondingly, the variable x is *decreasing* in T and *increasing* in σ if $x < 0$ and *increasing* in T and *decreasing* in σ if $x > 0$. Also, it is always *decreasing* in $\tilde{r}$. This implies the following.

- If there exist parameter values that imply $x < 0$, then x is minimised by selecting the maximum values of T and $\tilde{r}$ and the minimum value of σ.

- If there exist parameter values that imply $x > 0$, then x is maximised by selecting the maximum value of T and the minimum values of $\tilde{r}$ and σ.

99. Typical values for the annual risk-free rate of return $\tilde{r}$ are currently near-zero, but greater than -0.5%, which serves as the lower bound for the present exercise. Over the last 30 years the risk-free rate has ranged up to around 10% per annum. Annualised volatilities on (unleveraged) financial assets such as equities, bonds, commodities and indices typically range from the order of 10% to 30%. Finally, based on known typical terms for binary options given public information from providers, a range of possible terms from 5 minutes to 30 days is considered.

100.   Converting from an annual to a daily basis for the variables $\tilde{r}$ and σ, and expressing time in days, yields the following ranges for the BSM parameters for the purpose of this robustness test.

$$T \in [0.00139 , 30]$$

$$\tilde{r} \in [-0.0000198 , +0.0003968]$$

$$\sigma \in [0.006299 , 0.03149]$$

It follows that

$$\tilde{r} - \frac{1}{2}\sigma^2 \in [-0.000516 , +0.000377]$$

which establishes that there exist both positive and negative values of x for the region of parameters studied. Consequently, x is minimised by the maximum values of T and $\tilde{r}$ and the minimum value of σ, while it is .maximised by the maximum value of T and the minimum values of $\tilde{r}$ and σ

101.   The following estimated bounds can be obtained for the variable x, where the estimated minimum is denoted x_{max} and the estimated minimum is denoted x_{min}.

$$x_{max} = \frac{0.0000198 + 0.5 \times 0.006299^2 \ \sqrt{30}}{0.006299} = +0.0431$$

$$x_{min} = \frac{-0.0000198 + 0.5 \times 0.006299^2 \ \sqrt{30}}{0.006299} = -0.3278$$

102.   These values imply that for a comprehensive range of relevant parameter values, the estimated range of probabilities of finishing in the money for an up-down option is

$$[N(-x_{max}), N(-x_{min})] = [48.6\% , 62.8\%]$$